When Transparency Backfires:
Strategic Obfuscation and Financial Fragility*

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Abstract

We study delegated asset management in which a fund manager, privately informed of her skill, strategically obfuscates portfolio disclosures. Full transparency induces managerial reputation concerns, leading to excessively risky trading and lower returns. Obfuscation serves as a commitment device to mitigate these distortions, benefiting both the manager and investors. At intermediate obfuscation, self-fulfilling equilibria emerge: one with efficient trading and another with excessive risk-taking, generating fragility. Since skilled managers benefit more from obfuscation, optimal obfuscation increases with skill, yielding a non-monotone obfuscation-fragility relationship. Consequently, transparency mandates can backfire, reducing investor returns and pushing the market toward fragile equilibria.

JEL classification: G11, G12, G14, G23

Key words: informed trading, delegated asset management, obfuscation, reputation, financial fragility

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1 Introduction

Obfuscation is common in fund disclosures to investors.[1] Even when pursuing simple strategies, fund managers describe them in markedly different and often unnecessarily complex ways. For example, as deHaan, Song, Xie, and Zhu (2021) document, two funds that both track the S&P 500 describe essentially identical objectives with strikingly different clarity: Schwab states in a single sentence, "The fund's goal is to track the total return of the S&P 500 Index," while Deutsche describes it in 60 words, embedding the same objective in technical language.[2] Similar patterns arise in fund names: labels such as "Dynamic Opportunity Fund," "Flexible Growth Strategy," or "Opportunistic Equity Fund" convey little concrete meaning to non-professional retail investors at first glance, yet in practice often correspond to relatively simple large-cap portfolios that closely resemble the S&P 500. This pattern is puzzling, as standard signaling theory suggests that managers, seeking to convey skill and attract capital, should prefer greater transparency rather than opacity. Why would managers make information harder for their own investors to interpret? Perhaps more surprisingly, why would rational investors continue to allocate capital to funds that deliberately obscure their strategies? What are the incentives and market consequences when fund obfuscation becomes a manager's strategic choice?

To address these questions, we study an equilibrium model of financial market trading with delegated asset management. A fund manager, privately informed about her skill, raises capital from investors through obfuscated communications and trades a risky asset in the financial market à la Kyle (1985). We define skill as the manager-specific ability to uncover more information about asset payoffs than the market: the more skilled the manager, the larger the mispricing she can identify and the greater the trading profits she can potentially earn. To attract fund flows, the manager privately communicates information about her risky position (portfolio) to investors. Investors, in turn, use this fund information to infer the manager's skill and decide on capital allocation to the fund. Importantly, the manager strategically chooses the readability of

[1] Following Ellison and Ellison (2009), the term "obfuscation" refers to general practices that reduce the readability of information and distort communications. Accordingly, "transparency" and "opacity" refer to the degree of interpretability and precision of information with which investors can assess the fund's positions, rather than mere disclosure.

[2] Deutsche's S&P 500 index fund is described as "The fund seeks to provide investment results that, before expenses, correspond to the total return of common stocks publicly traded in the United States, as represented by the Standard & Poor's 500 Composite Stock Price Index (S&P 500 Index). The fund invests for capital appreciation, not income; any dividend and interest income is incidental to the pursuit of its objective."

this communication: by obfuscating information, she can make it more difficult for investors to back out her underlying skill.

The model yields two stable, self-fulfilling equilibria regarding the manager's trading strategy. One of the equilibria is similar to that of Kyle (1985), where the manager employs the profit-maximizing trading strategy. In the other, which we call the "reputation-driven" equilibrium, the manager trades the risky asset more aggressively in an attempt to appear skilled and attract large fund flows. This strategy increases trading volume, price volatility, and market liquidity but undermines the fund's expected performance by revealing too much private information to the market maker. Intuitively, the manager can boost investors' perception of her skill by placing a large order, either buy or sell, because such an action makes her appear as a skilled manager who has identified an asset with significant mispricing. In equilibrium, investors accurately anticipate the manager's strategy, so their learning is not manipulated. Nevertheless, the manager takes excessive risk to conform to investors' belief that the manager trades more aggressively than the profit-maximizing level, since deviating would cause them to (incorrectly) infer a less skilled manager and reduce fund flows. Key to our multiple equilibria lies in the manager's conflicting motives for revealing skill: on one hand, she wishes to hide her skill from the market maker to earn trading profits, but on the other hand, she wants to show it off to investors, aiming to establish high reputation and attract large fund flows.

The relative strength of these competing motives is governed by the manager's obfuscation. When the manager heavily obfuscates fund communications, investors can extract little information and therefore do not rely on these communications to infer skill. Thus, flows are insensitive to the manager's trading strategy, the showing-off motive is suppressed, and only the Kyle-like equilibrium survives. When the obfuscation level is low, in contrast, investors rely heavily on fund information, and only the reputation-driven equilibrium exists. Under intermediate obfuscation, however, both equilibria coexist, and which one prevails is determined by investors' self-fulfilling beliefs. If investors expect aggressive, reputation-driven trading, they treat fund information as a reliable signal of skill, making flows highly responsive to reported positions. Such responsiveness strengthens the showing-off motive and induces the manager to trade aggressively, confirming the investors' beliefs. The symmetric argument sustains the Kyle-like equilibrium when investors expect profit-maximizing behavior. A mere shift in beliefs, unaccompanied by any change in fundamentals, can therefore trigger a transition from the Kyle-like to the reputation-driven equilibrium where price volatility is high and trading performance is poor. We interpret this susceptibility to non-fundamental belief shifts as *fragility*.[3]

Building on these results, we endogenize obfuscation and study which types of managers contribute to financial fragility. By obfuscating fund information, a manager weakens investors' responsiveness to reported positions and curbs her showing-off incentive at the portfolio-decision stage. Although, when deciding on her risky position, she takes investors' belief-updating rule as given and is tempted to over-trade in an attempt to attract flows, she can *ex-ante* influence that rule through obfuscation. Obfuscation thus serves as a commitment device: by reducing transparency, the manager preempts perception-driven distortions and mitigates excessive risk-taking. This increases the profitability of her risky positions and improves the fund's expected performance, ultimately benefiting both herself and investors.

The value of this commitment is greater for higher-skilled managers, as they benefit more from improved profit margins. Therefore, more skilled managers optimally choose higher levels of obfuscation. This result is consistent with the "brain drain" documented by Kostovetsky (2017), whereby top mutual fund managers migrate to more opaque hedge funds and deliver superior performance. Crucially, the relationship between obfuscation and fragility is non-monotone. While heavily obfuscated funds yield a unique Kyle-like equilibrium and transparent funds yield a unique reputation-driven equilibrium, it is at intermediate levels of obfuscation where multiple equilibria, and thus fragility, arise. Because intermediate-skilled managers optimally choose intermediate levels of obfuscation, they expose financial markets to belief-driven instability. Such behavior is consistent with momentum crashes and sudden reversals observed during financial crises (Brunnermeier, 2009), as well as the abrupt strategy unwinding among quantitative funds documented by Khandani and Lo (2011). Importantly, fragility in our model is driven entirely by shifts in market beliefs and can arise without fundamental shocks, in line with the argument that large historical price swings are rarely accompanied by proportionally large changes in fundamentals (Gennotte and Leland, 1990). A direct policy implication follows: transparency mandates may push high-skilled managers into the fragile intermediate-obfuscation region, thereby *increasing* rather than decreasing systemic fragility.

[3] As in Greenwood and Thesmar (2011), we define a market as fragile if it is susceptible to non-fundamental demand shifts, such as those caused by changes in investor beliefs.

More broadly, by constraining obfuscation activities across all managers, such mandates exacerbate reputation-driven distortions and induce excessive risk-taking that lowers expected returns for both managers and investors. Transparency, in this sense, can backfire.

Our paper contributes to the understanding of obfuscation and opacity in financial markets. Empirically, Edelen, Evans, and Kadlec (2012), Badoer, Costello, and James (2020), and deHaan, Song, Xie, and Zhu (2021) document that mutual funds strategically obfuscate disclosures of their strategies and fee structures.[4] Theoretically, Carlin (2009), Carlin and Manso (2011), and Ellison and Wolitzky (2012) endogenize obfuscation as a means for firms to exploit consumers, and Sato (2014) studies opacity in hedge fund markets. We differ on two fronts. First, in our model, the manager's obfuscation is driven by reputation concerns vis-à-vis fund investors and serves as a commitment device that benefits both the manager and her investors, rather than as a rent-extraction tool. Second, our framework generates belief-driven multiple equilibria and financial fragility, which are absent in these earlier studies.

Our paper is also related to the literature on fragility and trading frenzies in financial markets. Cespa and Vives (2015) find multiple equilibria arising from short-term investor horizons and persistent liquidity trading, and Froot, Scharfstein, and Stein (1992) likewise emphasize strategic complementarities among short-term traders. Unlike these studies, our model does not feature short horizons; fragility instead arises from managers' concerns about fund flows. While Veldkamp (2006) and Goldstein, Ozdenoren, and Yuan (2013) generate trading frenzies through strategic complementarities in information markets or real-investment feedback, our reputation-driven equilibrium arises because managers trade aggressively out of fear of being perceived as less skilled by investors who expect aggressive trading.[5]

Lastly, our work contributes to the broader understanding of asymmetric information in financial markets. A large literature studies risk-taking when managerial ability is private but inferred through performance signals, including Huberman and Kandel (1993), Huddart (1999), DiMaggio (2015), Bijlsma, Boone, and Zwart (2018), and Malliaris and Yan (2021); we extend this line by deriving asset-pricing implications and managers' incentive to obfuscate. Our paper is closely re-

[4] Célérier and Vallée (2019) show that sellers of structured financial products deliberately engineer complexity into their offerings, suggesting that strategic obfuscation is a common practice among sellers of financial products more broadly.

[5] Cespa and Foucault (2014) also generate liquidity crashes from non-fundamental sources through cross-asset learning, though the mechanism differs from ours.

lated to Prat (2005), who shows in a general principal-agent setting that transparency can distort agents' actions away from principals' incentives, paralleling our result that transparency triggers showing-off trading.[6] However, Prat (2005) abstracts from financial markets and treats opacity as the principal's strategic choice; our model instead derives asset-pricing implications and fragility, and analyzes obfuscation incentives from the fund manager's side.[7] Gervais and Strobl (2020) endogenize managers' transparency choices but under exogenous costs; we instead identify endogenous opportunity costs of transparency, namely, reputation-driven trading distortions, so that transparency not only sorts manager types but can generate belief-driven multiple equilibria and market fragility.

The rest of the paper proceeds as follows. Section 2 presents a baseline model with an exogenous level of obfuscation, and Section 3 studies the equilibrium. Section 4 explores the manager's strategic obfuscation. The Appendix contains all proofs for the theoretical results.

2 Model

This section presents a one-period trading model à la Kyle (1985) with delegated asset management.

2.1 Setting

The economy consists of a *fund manager*, a competitive *market maker*, a *noise trader*, and $n \geq 2$ of *investors*. All players are risk neutral. The manager leverages her skill (defined below) to identify a single risky asset with potential mispricing, raises capital from investors, and allocates it between the risky asset and a risk-free asset with a zero interest rate.[8]

[6] Fishman and Hagerty (1995), John and Narayanan (1997), and Huddart, Hughes, and Levine (2001) show that exogenous mandatory disclosure rules induce informed traders to trade less informatively. In our model, transparency is endogenously chosen through the manager's obfuscation, and communications are targeted at investors. Unlike the literature, the manager trades more aggressively under transparency, and financial fragility arises. While disclosure in the literature is implicitly investor-protective, our model demonstrates that obfuscation can benefit both the manager and the investors.

[7] Dasgupta and Prat (2006), Dasgupta and Prat (2008), and Guerrieri and Kondor (2012) study career concerns with endogenous asset prices, finding volatility amplification, churning, and herding respectively, but do not analyze fragility or endogenous fund obfuscation by managers.

[8] We abstract away from information acquisition by individual investors and assume that they invest through a manager due to the lack of informational advantages. This assumption would be supported by, for example, information- or skill-acquisition costs, where only the fund manager acquires the skill at lower costs due to economies of scale (e.g., Gârleanu and Pedersen, 2018).

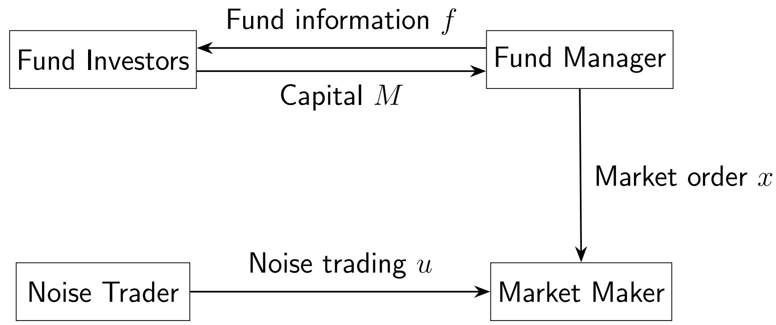


Figure 1: Capital Allocation and Trading

2.1.1 Skill

The manager can uncover more information about the asset's payoff than the market maker, and we interpret it as her *skill.* At the beginning of the period, the manager draws private information, s, from a normal distribution with mean zero and variance ω_s. Leveraging this information, she identifies an asset with payoff $\delta = \bar{\delta} + s$, where $\bar{\delta}$ is a publicly known mean payoff. Since the market maker does not observe s, her prior expectation of the asset's payoff is $\mathrm{E}[\delta] = \bar{\delta} + \mathrm{E}[s] = \bar{\delta}$, which is biased by s from the informed manager's perspective. As the market maker would set a price $p = \mathrm{E}[\delta] = \bar{\delta}$ in the absence of trades, s can be viewed as the asset's "potential mispricing."[9] Indeed, as shown later, the manager profits by buying the asset if $s > 0$ (underpriced) and selling it if $s < 0$ (overpriced); the larger the $|s|$, the larger the profit. We measure the manager's *realized* asset-selection skill by $|s|$, while the variance ω_s serves as an *ex-ante* measure of the average skill.[10,11]

2.1.2 Capital Allocation and Trading

We model the manager's portfolio choice and investors' capital allocation as a rational expectations equilibrium, as illustrated in Figure 1.

Manager's portfolio choice. Upon observing the signal s, the fund manager chooses the portfolio allocation between the risky and the safe asset. This action is represented by a market order for $x \in \mathbb{R}$ units of the risky asset, submitted to the market maker in the financial market. The manager

[9] s is not the *actual* mispricing because the market maker will take into consideration the presence of the informed manager; thus, p will deviate from $\bar{\delta}$ in equilibrium.

[10] This formalization of skill aligns with findings in the empirical studies that attribute fund managers' performance to stock-picking skills (e.g., Wermers, 2000; Kosowski, Timmermann, Wermers, and White, 2006).

[11] This definition follows from the fact that $\mathrm{E}[|s|]$ is proportional to $\sqrt{\omega_s}$.

decides on x by incorporating the reactions of the financial market (i.e., the asset price) and the capital inflow from her investors, as described in the following.

Investors' capital allocation. Upon observing fund information f (defined below), each investor chooses the amount of capital to invest into the fund. Investor i's capital allocation is denoted by $m_i \geq 0$, and the total fund flow or the assets under management (AUM) of the fund is defined by $M \equiv \sum_{i=1}^{n} m_i$. The fund's fee structure is proportional to the total fund proceeds: the manager takes a $\phi \in (0, 1)$ fraction of the proceeds, while the remaining $1 - \phi$ fraction is distributed to investors in proportion to their contribution to the fund, that is, m_i/M.[12]

Financial market. In the financial market, the market maker sets the asset price, p, based on the aggregate order flow, $x + u$, where u represents a random market order from the noise trader and follows a normal distribution with mean zero and variance ω_u. Due to competition, the market maker sets a semi-strong efficient price conditional on $x + u$:[13]

$$p = \mathrm{E}[\delta | x + u]. \tag{2.1}$$

2.1.3 Obfuscation

To communicate with investors, the manager disseminates a private noisy signal about her trading strategy to investors, defined as $f = x + \eta$, where η represents a normally distributed noise term with mean zero and variance ω_η.[14] This signal is referred to as the *fund information*, and its noise variance ω_η is used as the measure of *obfuscation*. It describes intentional obfuscation of the information that investors receive about the manager's true trading strategy. A higher ω_η means that the fund information is more complex, making the strategy harder to verify or reverse engineer. This reflects the idea that managers may deliberately obscure details by being vague, selective, or

[12] Introducing a fixed fee does not change our qualitative results, as long as the performance-based fee also exists. More generally, a common fee structure in the hedge fund industry ("2-and-20"-style) with separate performance fee ϕ_π on the realized profits and management fee ϕ_M on M can be incorporated into our model. This generalization only modifies the flow-performance sensitivity (defined below) and the fund-flow weight in the manager's objective, leaving all qualitative results unchanged.

[13] Assume that, on the equilibrium path, one competitive market maker trades the asset. As in Kyle (1985), competitively many market makers exist off the equilibrium path, ensuring the break-even price in the equilibrium.

[14] We assume that f is privately communicated between the manager and investors and is not observable to the market maker. Allowing the market maker to observe a noisy signal about x, however, does not affect the main qualitative results, as long as this signal is conditionally independent of f.

technically complex in their descriptions, while still engaging with investors to raise capital and to comply with disclosure rules. Since the manager chooses her trading strategy x based on the private signal s, the fund information f is informative about the manager's skill and the fund's return. Thus, fund investors use f to decide on their capital allocations. In what follows, we first consider the baseline model with exogenous ω_η, while Section 4 endogenizes it by allowing the manager to strategically obfuscate.

2.1.4 Maximization Problems and Equilibrium

The fund's payoff from the risky investment is δx, while the fund allocates $M - px$ to the risk-free asset. Hence, by denoting the return from the risky investment as $\pi = (\delta - p)x$, the total fund proceeds are represented as

$$y = \pi + M. \tag{2.2}$$

Given the realized skill s, the fund manager maximizes her expected fee income by controlling her investment strategy x:

$$\max_x \mathrm{E}\left[\phi y | s\right]. \tag{2.3}$$

Conditional on the fund information f, investor i chooses capital allocation m_i to maximize her expected return:[15]

$$\max_{m_i \geq 0} \mathrm{E}\left[\frac{m_i}{M}(1-\phi)y - m_i | f\right], \tag{2.4}$$

where the first term represents the expected payment from the fund, and the second term is the cost of capital.

Definition 1. *The equilibrium is defined by the market maker's pricing strategy p, the fund manager's trading strategy x, and investors' capital allocations $\{m_i\}_{i=1}^n$, such that, (i) p satisfies (2.1) given $x + u$, (ii) x solves (2.3) given s, p, and $\{m_i\}_{i=1}^n$, and (iii) for all i, m_i solves (2.4) given p, f, and $\{m_j\}_{j \neq i}$.*

[15] The fund manager may have an incentive to make a false report about y to her investors. Namely, by reporting $\tilde{y}$, she pays out $(1-\phi)\tilde{y}$ to investors and captures $y - (1-\phi)\tilde{y}$ on her own. To prevent this intentional misreporting, we assume that each investor can verify x after the trading stage. Alternatively, we may assume costly verification: investors collectively pay the verification cost C to back out x from the fund information in the end of the trading period. As long as C positively depends on the degree of fund obfuscation ω_η, our qualitative results remain the same.

3 Equilibrium

In this section, we search for a linear equilibrium in which the manager's market order x is linear in her skill s, and the market maker's pricing strategy p is also linear in the order flow $x + u$. Proofs for the analytical results are provided in the Appendix.

3.1 Trading Strategy

We conjecture and later verify that the manager's equilibrium trading strategy is

$$x(s) = \beta s, \tag{3.1}$$

where $\beta > 0$ is an endogenous constant determined in the equilibrium. Conjecture (3.1) states that the manager buys (sells) the asset if her s is positive (negative), where the order size is proportional to her skill $|s|$; the more skilled the manager, the more aggressively she trades.[16] The scale factor β is referred to as the *trading intensity*, representing how intensively the manager leverages her informational advantage over the market maker.

3.2 Execution Price

The market maker decides on the price of the asset by extracting information about the asset's payoff from the aggregate order flow $x + u$. Anticipating the trading strategy (3.1), the price in equation (2.1) can be rewritten as

$$p = \bar{\delta} + \lambda(x + u), \tag{3.2}$$

where

$$\lambda \equiv \frac{\beta \omega_s}{\beta^2 \omega_s + \omega_u} \tag{3.3}$$

is referred to as "Kyle's lambda," a measure of the price impact of order flow.[17] Following the literature, we define a market to be *liquid* if λ is small.

[16] This conjecture follows from the original Kyle model where an informed trader trades based on the difference between her belief and that of the market maker, i.e., $\mathrm{E}[\delta|s] - \mathrm{E}[\delta] = s$.

[17] Since the market maker believes that the manager follows (3.1), the linear filtering rule with normally distributed random variables yields $p = \mathrm{E}[\bar{\delta} + s|\beta s + u] = \bar{\delta} + \mathrm{E}[s] + \frac{\mathrm{Cov}[s, \beta s + u]}{\mathrm{Var}[\beta s + u]}(x + u - \mathrm{E}[\beta s + u]) = \bar{\delta} + \frac{\beta \omega_s}{\beta^2 \omega_s + \omega_u}(x + u)$.

3.3 Investors' Capital Allocation

By incorporating the fund proceeds $y = \pi + M$, investor i's optimization problem in (2.4) is

$$\max_{m_i \geq 0} \frac{m_i}{M}(1-\phi)\mathrm{E}\left[\pi|f\right] - \phi m_i. \tag{3.4}$$

Investor i solves this problem by incorporating the impact of her choice (m_i) on the total fund flow (M), while taking other investors' behavior ($\{m_j\}_{j\neq i}$) as given. As she increases her capital allocation, its marginal return diminishes due to the pro-rata allocation of the fund's proceeds, while the cost of capital linearly increases. This structure ensures the second-order condition of problem (3.4), and the first-order condition pins down the optimal capital allocation as follows:

$$m_i = \left(\frac{1-\phi}{\phi}\mathrm{E}[\pi|f]\sum_{j\neq i} m_j\right)^{\frac{1}{2}} - \sum_{j\neq i} m_j. \tag{3.5}$$

We focus on the symmetric equilibrium where all investors take the same action, $m_i = m$ for all $i = 1, 2, \cdots, n$. In this equilibrium, equation (3.5) reduces to

$$m = \frac{n-1}{n^2}\frac{1-\phi}{\phi}\mathrm{E}[\pi|f], \tag{3.6}$$

and the total fund flow becomes

$$M \equiv nm = \theta\mathrm{E}[\pi|f], \tag{3.7}$$

where $\theta \equiv \frac{n-1}{n}\frac{1-\phi}{\phi}$ represents the *flow-performance sensitivity*. Both the individual capital allocation and the total fund flow are linearly increasing in the expected trading profit conditional on investors' information, $\mathrm{E}[\pi|f]$. This is due to the performance-based fee structure, which makes the per-unit return to investors increasing in trading profits. The fund flow becomes more sensitive to the expected performance (θ increases) when the fee rate (ϕ) is low, as it reduces the cost of delegation and encourages investment; and when the fund has a large clientele (n), as it attracts capital from a larger number of investors.

3.4 Reputation

In determining the capital allocation in (3.5), each investor seeks to compute the expected fund profits $\mathrm{E}[\pi|f]$. This process, in turn, requires the investor to back out the manager's realized skill $|s|$ from fund information $f = x + \eta$.

Lemma 3.1. *Upon observing the fund information $f = x + \eta$, investors update their expectation about the fund's trading profit as*

$$\mathrm{E}\left[\pi|f\right] = (1-\lambda\beta)\beta\mathrm{E}[s^2|f] = (1-\lambda\beta)\beta\left(\hat{s}^2 + \hat{\omega}_s\right), \tag{3.8}$$

where the posterior expectation of s is

$$\hat{s} = \mathrm{E}[s|f] = \xi(x+\eta), \tag{3.9}$$

with

$$\xi = \frac{\beta\omega_s}{\beta^2\omega_s + \omega_\eta}, \tag{3.10}$$

and the posterior of the skill variance is

$$\hat{\omega}_s = \mathrm{Var}[s|f] = \frac{\omega_s\omega_\eta}{\beta^2\omega_s + \omega_\eta}. \tag{3.11}$$

The fund return $\pi = (\delta - p)x$ is a quadratic function of s, as both the manager's trading quantity x and the profit margin $\delta - p$ are proportional to s in expectation, which underlines equation (3.8). Therefore, investors seek to update their belief about the manager's skill based on fund information. Equation (3.9) describes investors' belief-updating process: $\hat{s}$ represents the updated expectation about skill and is referred to as the manager's *reputation* in the following discussion. According to equation (3.9), ξ measures how much investors rely on f in updating their belief and governs the sensitivity of the reputation to the fund's trading strategy x. Holding β constant, ξ is large when investors' prior is unreliable (ω_s is large) and the fund information is transparent (ω_η is small). More importantly, ξ itself also depends on the trading intensity β, as a more intensive trading strategy makes f more informative about s.

Equation (3.7) and Lemma 3.1 jointly imply that, on top of maximizing profits, delegated asset management generates an additional incentive for the manager to control her trading strategy x, as it affects the total fund flow M by influencing investors' expectations about fund returns.

3.5 Manager's Optimization

By incorporating the total fund flow ([3.7] and [3.8]), the manager's problem in (2.3) is rewritten as follows:

$$\max_{x} \; sx - \lambda x^2 + \theta(1 - \lambda\beta)\beta \left[\xi^2(x^2 + \omega_\eta) + \hat{\omega}_s\right]. \tag{3.12}$$

The first two terms represent the fee income from the investment return and are essentially identical to the trading profit in the original Kyle (1985) model. The last term with coefficient θ arises from the fee income associated with the fund flow M. It increases with the manager's risk taking, measured by x^2, because she can potentially inflate her reputation $|\hat{s}|$ by placing a large order (either buy or sell). The reaction of the expected fund flow to the trading strategy is influenced not only by the flow-performance sensitivity θ but also by the investors' updating coefficient ξ. This is because obfuscation ($\omega_\eta > 0$) prevents investors from perfectly inferring the manager's trading strategy from fund information. Importantly, investors' estimate of the manager's skill and, therefore, the reaction of the fund flow to the risky investment depend on the trading intensity β itself: the more intensively the manager trades, the more reliable the fund information becomes for investors. This structure is captured by the dependence of ξ on β in equation (3.10), which, as shown later, leads to the strategic complementarity between the manager's risk taking and investors' capital allocation based on reputation.

The unique solution to the manager's optimization problem in (3.12) (given β) is represented by

$$x = F(\beta)s, \tag{3.13}$$

where

$$F(\beta) \equiv \frac{1}{2\left(\lambda - \theta(1 - \lambda\beta)\beta\xi^2\right)}. \tag{3.14}$$

$F(\cdot)$ is a nonlinear function of β due to the dependence of λ and ξ on β.

3.6 Equilibrium

The manager optimally chooses (3.13) given that the market maker and investors believe that she follows strategy (3.1). In equilibrium, their beliefs must be correct, meaning that (3.1) and (3.13) must be consistent with each other. This belief consistency condition requires that

$$\beta = F(\beta). \tag{3.15}$$

That is, the equilibrium levels of β are determined at the fixed points of $F(\cdot)$. Having identified β, the equilibrium levels of x, p, and M are determined by equations (3.1), (3.2) and (3.7), respectively. In what follows, we first illustrate the determination of equilibrium β through a numerical example, followed by an analytical proposition that formally establishes the equilibria.

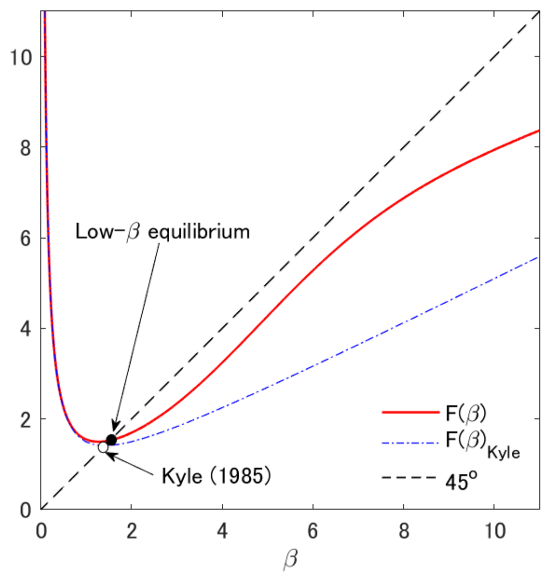
3.6.1 Fund Obfuscation and Multiple Equilibria

Figure 2 illustrates the determination of β for three different levels of obfuscation ω_η. The intersections of $F(\beta)$ (the solid line) and the 45-degree line yield the equilibrium levels of β. We also plot $F(\beta)_{\text{Kyle}}$ in the dash-dotted line, representing the original Kyle (1985) model. This benchmark case arises when the level of obfuscation is infinite (i.e., $\omega_\eta \to \infty$), as it makes the fund flow inelastic to the manager's trading strategy, and the delegated asset management becomes irrelevant.

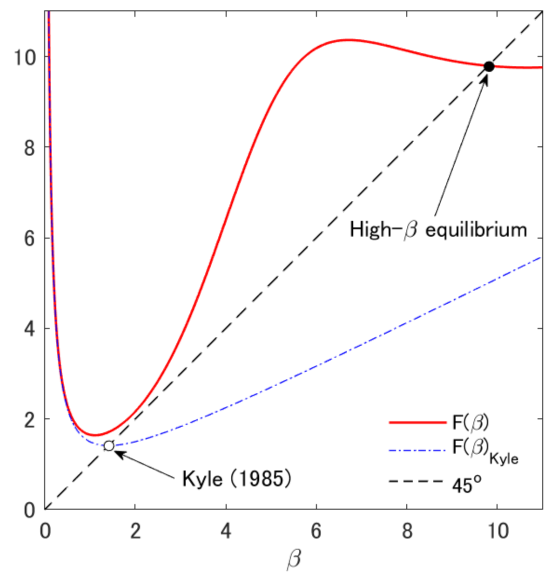
Panel (a) of Figure 2 shows that if ω_η is large, there is a unique "low-β equilibrium," in which β is close to the trading intensity without delegated asset management. Panel (b) shows that a small ω_η leads to a unique "high-β equilibrium," in which β is much larger than that in the original Kyle (1985) model. Panel (c) shows that multiple equilibria arise for an intermediate level of ω_η. Specifically, the high-β and the low-β equilibria are stable, as $F(\beta)$ crosses the 45-degree line from above, whereas the one with intermediate β is unstable, as $F(\beta)$ crosses the line from below.[18] In what follows, we focus only on the stable equilibria.

To see the intuition behind the different equilibrium patterns presented in Figure 2, note that the fund manager faces two conflicting motives when trading. On the one hand, she seeks to avoid

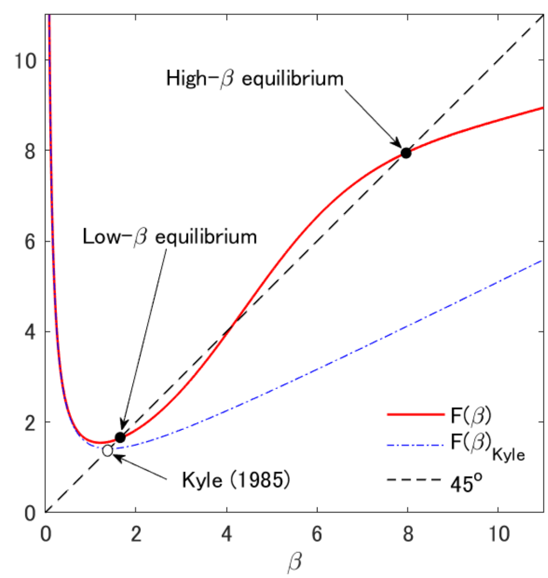
[18]We define an equilibrium to be stable (unstable) if the corresponding β is a stable (unstable) fixed point of $F(\cdot)$, that is, $|F'(\beta)| < 1$ (> 1). A stable equilibrium is robust to a small perturbation to the agents' belief about β, meaning that the economy converges back to the original equilibrium through the best-response dynamics of the agents' beliefs.



(a): High obfuscation level (large ω_η) $\Rightarrow$ unique low-β equilibrium.



(b): Low obfuscation level (small ω_η) $\Rightarrow$ unique high-β equilibrium.



(c): Intermediate obfuscation level (intermediate ω_η) $\Rightarrow$ multiple high-β & low-β equilibria.

Figure 2: Determination of β

Note: The figure plots $F(\beta)$, where the values of ω_η are 48, 28, and 38 in panels (a), (b), and (c), respectively. The equilibrium values of β are determined at the fixed points of $F(\cdot)$. The parameter values used in the figures are $\omega_s = 1$, $\omega_u = 2$, $n = 5$, and $\phi = 0.02$.

trading too aggressively on her private information, as doing so would move the price excessively and reduce her trading profits. As in Kyle (1985), this motive leads the manager to decrease β. On the other hand, she wishes to trade intensively to signal her skill to her investors, aiming to establish a strong reputation and attract large fund flows. This motive encourages her to increase β. In summary, she faces a tradeoff between "hiding" her skill from the market maker and "showing it off" to her investors. The degree of obfuscation ω_η governs this tradeoff by influencing the sensitivity of the manager's reputation held by investors to her investment behavior. In particular, Lemma 3.1 demonstrates that the fund flow M becomes less sensitive to x under high obfuscation (i.e., ξ is small) compared to the fully transparent case, as it becomes difficult for investors to estimate the skill based on the fund information f. Therefore, fund obfuscation diminishes the strategic complementarity between the manager's risk taking and the reputation-driven capital allocation by investors, thereby weakening the showing-off motive.

For a highly obfuscated fund with a large ω_η (panel [a]), the hiding motive dominates the showing-off motive, leading to a unique low-β equilibrium. In this equilibrium, investors believe that the manager's trading intensity is relatively low and the fund information is not very informative, making the reputation and the fund flow insensitive to x. Although the manager could inflate her reputation by placing an order larger than what her investors anticipate, she refrains from doing so, because it would cause a large price impact and reduce the fund's investment profits, while fund flows and her fee income would not increase much due to a small ξ. Consequently, the equilibrium β is close to the trading intensity without delegated asset management, meaning that it is a "Kyle-like equilibrium."

For a transparent fund with a small ω_η (panel [b]), the showing-off motive prevails, resulting in a unique high-β equilibrium. In this equilibrium, investors believe that the manager is trading aggressively. The reputation and fund flows are sensitive to the manager's trading strategy through the fund information. Under such beliefs, the manager is actually willing to trade aggressively, despite the price impact that undermines her investment profits. Importantly, investors' learning is not manipulated, as their belief about the manager's trading strategy is correct. Nonetheless, the manager places a large order because doing otherwise would induce her investors to incorrectly estimate that she is less skilled, thereby deteriorating her reputation. She trades aggressively solely to conform to her investors' belief and to avoid a further decline in fund flows. To emphasize this mechanism, we call this equilibrium the "reputation-driven equilibrium."

For intermediate levels of ω_η (panel [c]), the strategic complementarity between investors' reputation-based capital allocation and the manager's risk taking is neither weak nor strong, and there is no clear dominance between the two motives. Consequently, both the low-β and the high-β equilibria are supported under intermediate levels of obfuscation. Each equilibrium is associated with a self-fulfilling belief of the market maker and investors.[19]

3.6.2 Equilibrium Characterization

The following proposition summarizes the above discussion and provides the analytical characterization of the equilibria.

Proposition 3.1 (Equilibrium characterization)**.** *There exists a critical value of the flow-performance sensitivity, denoted as θ_0.*

(i) If $\theta \leq \theta_0$, the equilibrium is unique and characterized by a threshold of the obfuscation level ω_0, such that, $\omega_\eta \leq \omega_0$ leads to the high-β equilibrium and $\omega_\eta \geq \omega_0$ leads to the low-β equilibrium.

(ii) If $\theta > \theta_0$, there are two thresholds of fund obfuscation, ω_L and $\omega_H (> \omega_L)$. When $\omega_\eta \leq \omega_L$, the high-$\beta$ equilibrium is unique; when $\omega_\eta \geq \omega_H$, the low-$\beta$ equilibrium is unique; and when $\omega_\eta \in (\omega_L, \omega_H)$, the high- and low-$\beta$ equilibria coexist.

Figure 3 illustrates the result in Proposition 3.1. It indicates that both fund obfuscation ω_η and flow-performance sensitivity θ are critical in determining the equilibrium type, as they jointly influence the reaction of the fund flow to changes in the manager's trading strategy, thereby affecting her showing-off motive. As high levels of θ strengthen this motive, we obtain the following corollary:

Corollary 3.1. *(i) When $\theta \leq \theta_0$, the threshold of fund obfuscation ω_0 is monotonically increasing in θ with $\lim_{\theta \to 0} \omega_0 = 0$.*

(ii) When $\theta > \theta_0$, both ω_L and ω_H are increasing in θ. Moreover, the region for multiple equilibria, (ω_L, ω_H), expands as θ increases.

For a given level of fund obfuscation ω_η, a weak flow-performance sensitivity supports only the low-β equilibrium, as the hiding motive strictly dominates the weak showing-off motive. However,

[19] For simplicity, we assume that agents do not agree to disagree, so that they coordinate their beliefs when parameter values are such that multiple equilibria exist.

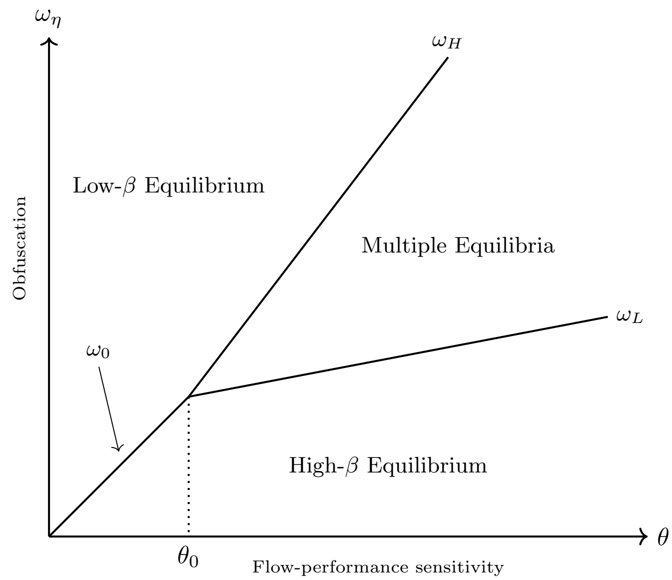


Figure 3: Equilibrium Characterization

Note: This figure illustrates the equilibrium states using the flow-performance sensitivity (θ) and the level of fund obfuscation (ω_η). When $\theta \leq \theta_0$, the cutoffs of ω_η converge to $\omega_H = \omega_L = \omega_0$.

as θ increases, the showing-off motive strengthens, leading the manager to trade more intensively, and the high-β equilibrium emerges. The two motives are balanced and generate multiple equilibria when the fund is moderately obfuscated and fund flows are sensitive to the expected investment performance. Otherwise, one of the conflicting motives dominates the other, making either the Kyle-like or the reputation-driven equilibrium the unique outcome.

3.7 Effects of Obfuscation

3.7.1 Financial Market Outcomes across Equilibria

Comparing the two stable equilibria, does the manager perform better in one of them? What are the implications for market liquidity, asset prices, and learning about asset-selection skills?

Corollary 3.2 (High-β vs. low-β equilibrium)**.** *In the high-β equilibrium, relative to the low-β equilibrium, the following results hold.*

(i) The market is more liquid, that is, λ is lower;

(ii) The manager's expected investment performance given the realized skill, $\mathrm{E}[(\delta - p)x|s] =$

$\frac{\beta\omega_u s^2}{\beta^2\omega_s+\omega_u}$, *is lower;*

(iii) The price informativeness, defined by $\frac{1}{\mathrm{Var}[\delta|p]} = \frac{\beta^2\omega_s+\omega_u}{\omega_u\omega_s}$, *is higher;*

(iv) The price volatility, $\mathrm{Var}[p] = \frac{\beta^2\omega_s^2}{\beta^2\omega_s+\omega_u}$, *is higher; and*

(v) Investors' estimate of the manager's skill is more precise, that is, $\hat{\omega}_s = \mathrm{Var}[s|f] = \frac{\omega_s\omega_\eta}{\beta^2\omega_s+\omega_\eta}$ *is lower.*

Statement (i) of Corollary 3.2 holds because price impact λ is decreasing in β in the equilibrium. Namely, the information in the order flow is exaggerated in that the order size is too large relative to the true $|s|$ due to the showing-off motive. Thus, the market maker attempts to partially "undo" the manager's information revelation by incorporating less of it into the price, resulting in a weaker price impact.[20] Since the high-β equilibrium emerges only when obfuscation is limited, the statement also implies that greater obfuscation leads to lower market liquidity.

The intuition for statement (ii) is closely related to that for statement (i). In the high-β equilibrium, the manager earns a lower trading profit due to a smaller informational advantage over the market maker. This result may appear inconsistent with statement (i) because, ceteris paribus, a lower price impact is typically associated with a higher trading profit. However, the manager's order size $|x|$ in the high-β equilibrium is so large that the *overall* price reaction (i.e., λ multiplied by $|x|$) is larger than that in the low-β equilibrium. Consequently, the manager moves the price excessively and earns a lower trading profit in this equilibrium. Due to her aggressive trading, more information about s, which is informative about δ, is incorporated into the price in the high-β equilibrium, supporting statement (iii).

Statement (iv) follows, again, from the manager's aggressive trading in the high-β equilibrium. Specifically, she places a large buy (sell) order when $s > 0$ ($s < 0$), even if $|s|$ is small. This causes a large upward (downward) pressure on p, leading to greater price variability. This result aligns with the insight of Guerrieri and Kondor (2012) that reputation concerns amplify price volatility, although their mechanism differs from ours.[21]

In the high-β equilibrium, the manager reveals more information about s, thereby improving

[20] λ represents the price's reaction to the order flow $x + u$, rather than its reaction to the manager's private information s. The price impounds s with coefficient $\beta\lambda$, which is monotonically increasing in β.

[21] Guerrieri and Kondor (2012) argue that bond price volatility increases because fund managers demand a premium to offset the risk of reputation damage in the event of default.

the precision of investors' estimation about the manager's skill, as statement (v) demonstrates. This makes fund flows more responsive to skill, as investors can assess it more accurately.

3.7.2 Obfuscation, Trading Style, and Performance

Having compared the two stable equilibria at a fixed level of ω_η, we now examine how obfuscation itself shapes equilibrium trading behavior, fund performance, and investor welfare.

Obfuscation and trading intensity. Proposition 3.1 yields novel implications connecting the level of obfuscation and trading styles.

Corollary 3.3. *Both in the high- and low-β equilibria, the trading intensity β is monotonically decreasing in the obfuscation level ω_η.*

Figure 4 illustrates the result in Corollary 3.3: the left panel represents the case with a unique equilibrium by setting a weak flow-performance sensitivity, while the right panel involves multiple equilibria for a middle range of ω_η due to a strong flow-performance sensitivity. First, as both panels illustrate, highly obfuscated funds tend to adopt low-intensity strategies (i.e., the low-β equilibrium) that minimize market impact, prioritizing profits over reputation. In contrast, transparent funds engage in high-intensity strategies with flashy risk taking (i.e., the high-β equilibrium), seeking reputation among investors. This relationship arises because the manager's reputation becomes insensitive to her trading behavior when the fund information is obfuscated and difficult to interpret for fund investors. The model predicts that these tendencies are more pronounced when the flow-performance sensitivity is weak, as the equilibrium is unique regardless of the obfuscation level (panel [a] of Figure 4).

These results are consistent with common observations in the real market. For example, quantitative hedge funds are often extremely opaque and highly profit-driven, focusing on consistent performance, with benefits from attracting fund flows being regarded as secondary to profitability (Schwager, 2012). On the other hand, some transparent funds, such as certain thematic ETFs or crypto-focused funds, engage in bold, attention-grabbing strategies to attract investor flows. These funds often emphasize visibility and narrative-driven positioning, which can lead to volatile returns and greater risk taking.

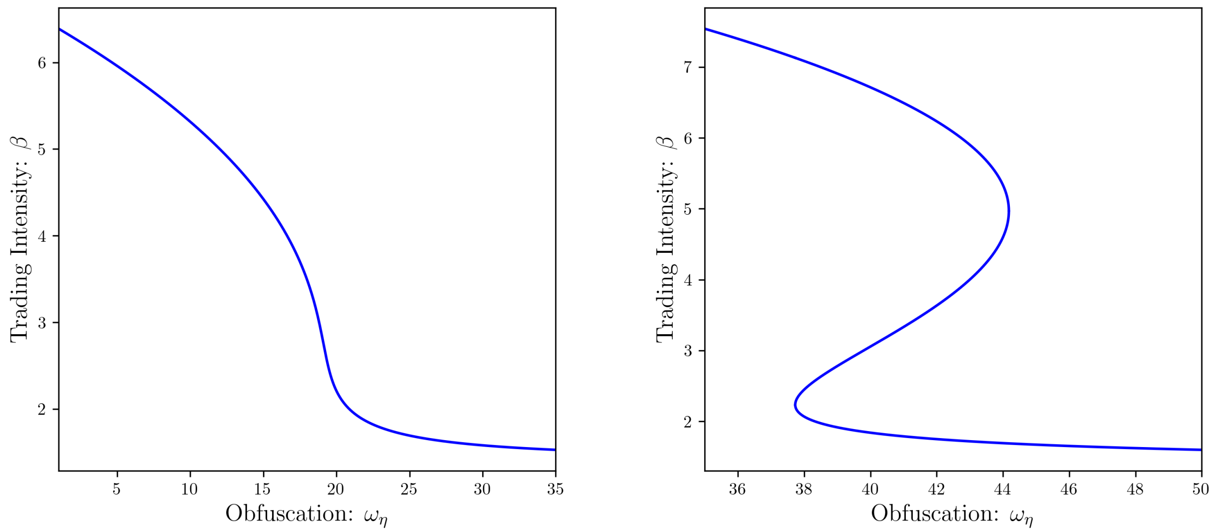
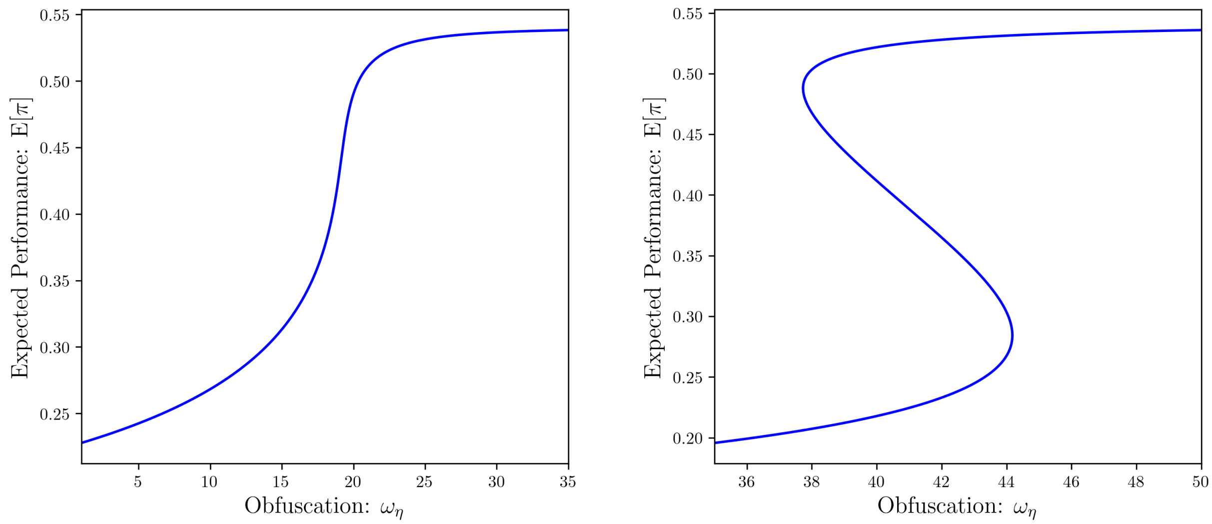


Figure 4: Obfuscation Level and Trading Intensity

Note: The figure plots the trading intensity β against the level of obfuscation ω_η with different values of the flow-performance sensitivity: $\omega_s = 2.0, \omega_u = 3.0, n = 2, \phi = 0.0475$ ($\theta = 10.0$) for the left panel and $\phi = 0.018$ ($\theta = 27.3$) for the right panel.

The second implication is drawn from the model's multiple equilibria, as panel (b) in Figure 4 illustrates. Namely, the relation between funds' activities (β) and their degrees of obfuscation (ω_η) becomes indeterminate when ω_η is at an intermediate level. Depending on market beliefs, funds with intermediate obfuscation may shift between profit-seeking and reputation-driven strategies, creating non-fundamental fluctuations. Our result suggests that even relatively opaque funds may follow the crowd due to market beliefs, switching between safe and risky strategies.

A notable observation in line with this result is momentum crashes (Daniel and Moskowitz, 2016), where funds engaging in risky and visible momentum trades often switch to safer strategies, generating sharp reversals and amplifying price declines. More generally, the rapid shifts between risk-taking and solid performance-based behavior have contributed to market fragility and volatility, as documented in various studies on economic crises, such as Brunnermeier (2009) on the Global Financial Crisis in 2007. Importantly, large fundamental shifts that support such transitions are rarely identified (Gennotte and Leland, 1990). Our model does not require such fundamental shocks, as a change in beliefs alone can trigger a shift.



(A) Low Flow-performance Sensitivity

(B) High Flow-performance Sensitivity

Figure 5: Obfuscation Level and Expected Performance

Note: The figure plots the unconditional expected performance of the fund $\mathrm{E}[\pi]$ against the level of obfuscation ω_η. Parameter values are the same as those used in Figure 4

Obfuscation and fund performance. We next turn to the relationship between the level of obfuscation and fund performance. Given the trading intensity β, the unconditional expectation of fund performance is computed as

$$\mathrm{E}[\pi] = \frac{\beta\omega_u}{\beta^2\omega_s + \omega_u}\omega_s. \tag{3.16}$$

As in the standard Kyle (1985) framework, the first fraction represents the profit margin that each unit of informational advantage would earn, and the second term ω_s represents the average magnitude squared of the informational advantage.

The obfuscation level ω_η does not directly show up in equation (3.16), but it influences the expected performance through the equilibrium trading intensity β, as Corollary 3.3 demonstrates.

Corollary 3.4. *Both in the low- and high-β equilibria, the expected fund performance* $\mathrm{E}[\pi]$ *is monotonically increasing in the obfuscation level ω_η.*

Figure 5 plots $\mathrm{E}[\pi]$ in relation to ω_η, which traces the reactions of β in Figure 4. In our model, holding other parameters fixed, $\mathrm{E}[\pi]$ declines with the trading intensity β. This arises from the presence of the showing-off motive. Although unconditional expected performance is maximized at the Kyle (1985) benchmark, which is driven solely by the hiding motive, the manager in our model trades more aggressively in an attempt to attract fund flows. It reveals too much private information to the market maker and lowers the fund performance, while the manager is inclined to deviate from the profit-maximizing strategy because, taking investors' inference rule as given at the trading stage, less aggressive trading would lower their assessment of her skill and reduce her expected fee income. Obfuscation, conversely, makes it harder for the investors to infer the manager's skill from her actions, thereby diminishing her showing-off motive. This leads her to pursue more profit-driven strategies, bringing β closer to the profit-maximizing level. As a consequence, obfuscation helps improve the expected fund performance.

Furthermore, a direct consequence of equation (3.7) is that the same monotonic effect carries over to the unconditional expected fund flow. Since $\mathrm{E}[M] = \theta\,\mathrm{E}[\pi]$ by the law of iterated expectations, $\mathrm{E}[M]$ is also monotonically increasing in the obfuscation level ω_η in both equilibria.

Obfuscation and investor welfare. Interestingly, investors' *ex-ante* expected utility is also increasing in the level of obfuscation. According to equation (3.6), each investor expects to obtain

$$U_i \equiv \frac{1-\phi(1+\theta)}{n}\mathrm{E}[\pi], \tag{3.17}$$

so that her expected utility is proportional to the fund's unconditional expected performance.

Thus, Corollary 3.4 demonstrates that even investors prefer obfuscated funds from an *ex-ante* perspective. This somewhat counterintuitive result provides a theoretical explanation for the seemingly puzzling real-world observation that investors willingly allocate substantial capital to opaque funds that deliberately obfuscate their investment strategies. The intuition is as follows. When the fund is transparent, investors can infer the manager's skill from her actions. Anticipating such scrutiny, the manager becomes concerned about investors' perceptions and engages in inefficient risk taking, thereby reducing fund performance. When obfuscation is high, however, investors' ability to infer the manager's skill is limited, which dampens her reputation concerns. This allows her to pursue more profit-driven strategies, improving fund performance. Consequently, obfuscation can be beneficial not only for the manager but also for investors.

Taken together, Corollaries 3.3 and 3.4, along with equation (3.17), deliver a cautionary message for transparency: an exogenous reduction in ω_η lowers $\mathrm{E}[\pi]$, deteriorating both the manager's and investors' *ex-ante* utility. Transparency, in this sense, can backfire. This observation motivates the endogenous-obfuscation analysis in Section 4, where we ask how much the manager herself would obfuscate.

3.8 Implications

This section discusses further economic implications drawn from the equilibrium analyses.

3.8.1 Financial Fragility

The self-fulfilling nature of multiple equilibria suggests that financial markets may become vulnerable to non-fundamental factors. A shift in investor beliefs alone, without any structural changes, could cause the market to transition from the standard Kyle-like equilibrium to the reputation-driven equilibrium, characterized by high price volatility and poor trading performance. We interpret this phenomenon as a form of *fragility* in financial markets.

Our model suggests that fragility may occur when flow-performance sensitivity (θ) is sufficiently high. This result is consistent with empirical studies documenting that strong flow-performance sensitivity can amplify trading pressure and contribute to market fragility by inducing managers to adjust trading behavior in response to investor flows rather than fundamentals (e.g., Coval and Stafford, 2007; Goldstein, Jiang, and Ng, 2017). While this literature typically attributes fragility to flow dynamics without explicitly focusing on the size of the investor base, our results suggest that a larger clientele (n) mechanically increases θ and thereby sows the seeds of market fragility.

A high flow-performance sensitivity can also arise when the fund management fee (ϕ) is relatively low. This result may appear to run counter to the recent debate on performance-based compensations for fund managers and advisory contracts, where payment structures that strongly reward performance are often criticized for encouraging excessive risk-taking. In our model, such performance-based compensation can actually stabilize the market by eliminating the reputation-driven equilibrium and guiding the economy toward a unique low-β equilibrium. This is consistent with the empirical finding by Dass, Massa, and Patgiri (2008) that high-powered advisory contracts do not induce investment in bubbly stocks but instead reduce such investments, highlighting the stabilizing effect of incentive compensations on financial markets.

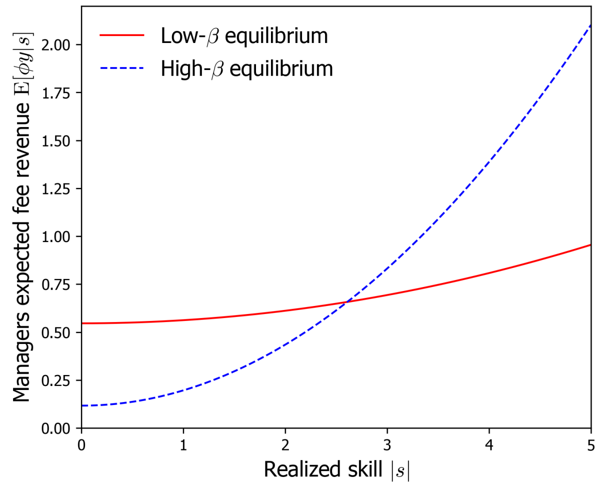


Figure 6: Skill versus Expected Fee Revenue

Note: The figure plots the manager's expected fee income at the low-β equilibrium (solid line) and high-β equilibrium (dashed line) for different levels of skill, $|s|$. The parameter values used in the figures are $\omega_s = 1$, $\omega_u = 2$, $\omega_\eta = 38$, and $\theta = 27$.

3.8.2 Compensation in Fund Management Industry

Our model provides insights into how skills are reflected in compensation in the fund management industry. As shown in statement (v) of Corollary 3.2, fund investors learn s more precisely in the high-β equilibrium than in the low-β equilibrium, as the manager demonstrates her skill by trading more intensively. Consequently, in the high-β equilibrium, the manager's expected compensation, $\mathrm{E}[\phi y|s]$, becomes more sensitive to her realized skill. As an illustration, Figure 6 plots the manager's expected compensation in relation to her skill in the low- and high-β equilibria. In the high-β equilibrium, investors' accurate evaluations result in greater pay inequality: low-skilled managers would receive very low compensation, while high-skilled ones are very highly paid. This result corroborates empirical observations, such as those provided by Philippon and Reshef (2012) and Böhm, Metzger, and Strömberg (2018), highlighting the comparatively large wage inequality in the fund management industry. Célérier and Vallée (2019) attribute the source of wage inequality to the scalability of skill, while inequality in our model arises from managers' incentives to attract fund flows through establishing high reputation.

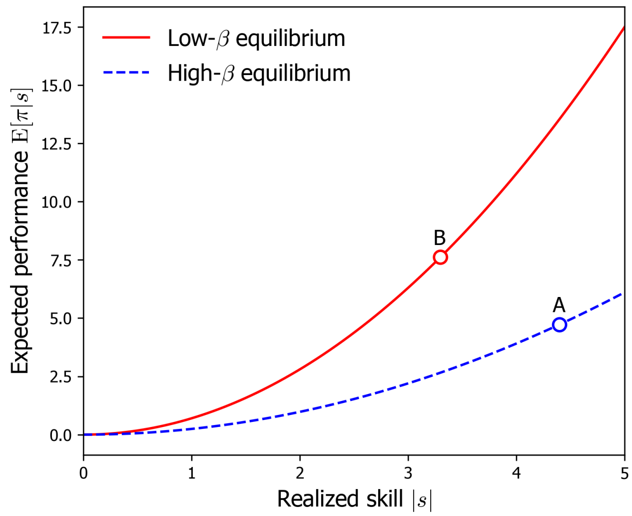


Figure 7: Skill versus Expected Trading Profit

Note: The figure plots the expected trading profit $\mathrm{E}[\pi]$ at the low-β equilibrium (solid line) and the high-β equilibrium (dashed line) for different levels of skill $|s|$. The parameter values used in the figure are $\omega_s = 1$, $\omega_u = 2$, $\omega_\eta = 38$, and $\phi = 0.02$.

3.8.3 Skill and Performance Persistence

Do high-skilled managers consistently perform well? Not always: skilled managers may not necessarily outperform less skilled ones. Consider a highly skilled manager and a relatively less skilled one. Since trading profits are increasing in managers' skill $|s|$, the former is expected to achieve higher trading profits than the latter *conditional on* both managers trading at the same intensity β. However, if their investors believe, for whatever reason, that the skilled manager invests more aggressively than the less skilled one, and managers conform to such beliefs, a high skill level does not translate into performance. This is because the skilled manager, trading at the reputation-driven equilibrium, reveals too much private information to the market maker. Hence, compared to the less skilled manager, who plays the Kyle-like equilibrium, the skilled manager may earn lower expected profits. Figure 7 illustrates such a situation.[22] It plots the expected trading profits (performance) at high- and low-β equilibria with different levels of skill, suggesting that a low-skilled manager in the low-β equilibrium (point B) outperforms a high-skill manager in the high-β equilibrium (point A).

The empirical literature (e.g., Jensen, 1967; Busse, Goyal, and Wahal, 2010) finds that asset

[22]The same argument holds for the comparison of the unconditional expected profit, $\mathrm{E}[\pi]$, using skill variance ω_s as an *ex-ante* measure of the expected skill.

managers do not outperform passive benchmarks on average and, in cross-section, those who outperform cannot do so consistently. While the literature attributes the lack of persistent performance to the lack of skill, our result proposes an alternative explanation. Several studies reconcile the existence of skill with the lack of persistent performance. Berk and Green (2004), for example, argue that skill-based alphas eventually disappear due to competition between investors and decreasing returns to scale at the fund level.[23] Our model contributes to the understanding of this matter by showing that the absence of a systematic skill-performance relationship could be the result of self-fulfilling multiple equilibria.

4 Strategic Obfuscation

This section analyzes endogenous obfuscation. Our results shed light on why managers obfuscate and which types of managers contribute to financial fragility.

4.1 Setup

Obfuscating fund information. Prior to investors' capital allocation and the manager's investment decision, the fund manager chooses the degree of obfuscation ω_η, anticipating the equilibrium reactions of investors and the market maker specified in Section 3. For example, in many mutual funds, managers provide information about their strategies to investors in accordance with regulatory requirements. However, they strategically design the way this information is presented and convey a deliberately uninformative description even when the underlying holdings are simple. We assume that such practices entail costs: the manager pays the obfuscation cost, $C(\omega_\eta)$ with $C'(\cdot) > 0$ and $C(0) = C'(0) = 0$, to increase ω_η. This cost can be interpreted as pecuniary costs arising from investments in communication and marketing resources to design abstract narratives and vague labels that replace direct descriptions of risky portfolios. It may also reflect non-pecuniary effort that managers expend in carefully structuring disclosure documents to maintain opacity while complying with regulations.

[23] Pastor, Stambaugh, and Taylor (2015) document increases in skill in finance and attribute the lack of corresponding increases in performance to decreasing returns to scale at the industry level.

Sunspot shock. To formalize the manager's optimization problem in the rational expectations framework, we introduce an equilibrium-selection device in cases of multiple equilibria. In particular, following the most common approach, we assume that market participants condition their actions on the realization of an exogenous *sunspot* shock, characterized by the binary random variable $z \in \{0, 1\}$.[24] Namely, whenever parameters support multiple equilibria, market participants play the high-β equilibrium if a spot appears on the sun ($z = 1$), whereas the low-β equilibrium is realized if no spots are observed ($z = 0$). Accordingly, the high- and low-β equilibria are realized with probability $\rho_H = \Pr(z = 1)$ and $\rho_L = 1 - \rho_H$, respectively. The shock is realized after the fund manager sets the obfuscation level but prior to the fund-raising stage.

Manager's expected utility. The manager controls ω_η to maximize her *ex-ante* expected utility derived from the fee income, net of the obfuscation cost C.

$$U = \mathrm{E}[\phi y] - C(\omega_\eta), \tag{4.1}$$

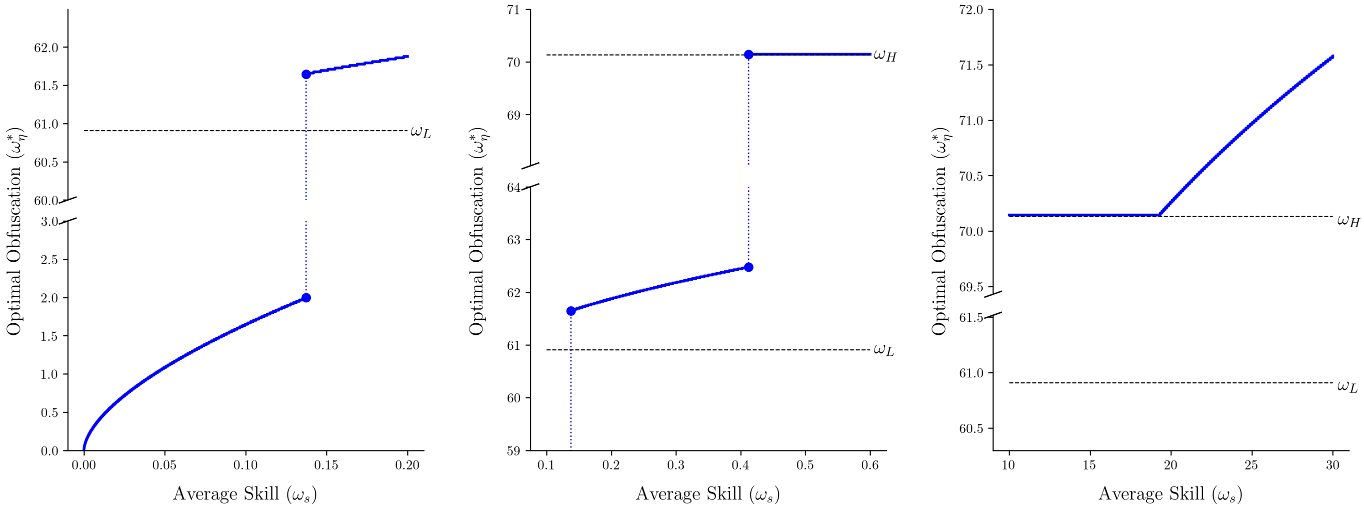
where the expectation operator incorporates the sunspot shock together with all other random variables introduced so far.

Equilibrium. In this extended model, the definition of equilibrium augments Definition 1 of the baseline model by additionally requiring that ω_η maximizes the manager's objective function U in equation (4.1).

4.2 Equilibrium Obfuscation

Equation (4.1) shows that the manager's expected utility depends on ω_η only through the unconditional expected performance $\mathrm{E}[\pi]$ and the obfuscation cost $C(\omega_\eta)$. From Corollary 3.4 in Subsection 3.7.2, obfuscation raises $\mathrm{E}[\pi]$ by preempting distortions arising from the manager's showing-off motive. The manager therefore uses obfuscation as a commitment device, trading off its benefit in mitigating these distortions against its cost.

[24]The sunspot equilibrium is proposed by Diamond and Dybvig (1983) as an equilibrium selection device in the context of bank runs and formalized in the equilibrium analyses by the subsequent studies, such as Cooper and Ross (1998).



(A) Low Average Skill

(B) Intermediate Average Skill

(C) High Average Skill

Figure 8: Average Skill and Optimal Obfuscation

Note: The figure plots the optimal degree of obfuscation ω_η^* against the manager's expected (average) skill ω_s, focusing on three regions that correspond to the equilibrium types. The dashed horizontal lines represent obfuscation thresholds, ω_L and ω_H, defined in Proposition 3.1. We use the parameter values $\phi = 0.03, n = 5, \omega_u = 5, \rho_H = 0.10$, and the obfuscation cost is $C = c\omega_\eta^\gamma$ with $c = 10^{-4}$ and $\gamma = 1.85$. For certain parameter configurations, the increasing region above ω_H is absent, and the optimal degree of obfuscation is pinned at ω_H for all sufficiently high skill levels. The qualitative features of the solution are nonetheless robust across a range of parameter values.

We now solve for the manager's optimal obfuscation, ω_η^*, that maximizes U in (4.1) and characterize how it varies with skill.

Proposition 4.1. *The optimal obfuscation ω_η^* is positive for all levels of the expected skill $\omega_s > 0$ and non-decreasing in ω_s. Moreover, ω_η^* is strictly increasing in ω_s whenever ω_η^* is interior to each equilibrium regime.*

Figure 8 presents a numerical illustration of Proposition 4.1, additionally revealing the discontinuous structure arising from equilibrium regime switches at ω_H and ω_L.

4.2.1 Skill, Obfuscation, and Fragility

Proposition 4.1 and Figure 8 show that the manager's optimal obfuscation ω_η^* is globally increasing in the average skill ω_s. The mechanism follows directly from the commitment role of obfuscation established in Corollary 3.4. A manager with higher ω_s expects to obtain a larger informational advantage and thus stands to gain more from an improvement in her profit margin. Since obfuscation mitigates the showing-off motive and steers the manager's strategy toward the profit-maximizing benchmark, the marginal benefit of obfuscation rises with ω_s. This positive skill–obfuscation relationship is consistent with the empirical pattern in which skilled managers disproportionately migrate to opaque investment vehicles and subsequently deliver superior returns (Kostovetsky, 2017).

Furthermore, the interaction between skill, obfuscation, and equilibrium selection generates a non-monotone relationship between obfuscation and financial fragility. As noted in Section 3, the equilibrium type is determined by the manager's obfuscation choice: transparent funds ($\omega_\eta < \omega_L$) support only the high-β equilibrium, funds with intermediate degrees of obfuscation ($\omega_L \leq \omega_\eta \leq \omega_H$) support multiple equilibria, and highly obfuscated funds ($\omega_\eta > \omega_H$) yield only the low-β equilibrium. Because optimal obfuscation is increasing in skill, this induces a systematic mapping from skill to fragility, such that managers with intermediate skill optimally choose intermediate obfuscation in $\omega_\eta \in [\omega_L, \omega_H]$, placing them in the multiple-equilibria region and making them a source of financial fragility. In contrast, high-skill managers heavily obfuscating their funds secure the unique low-β equilibrium with superior trading performance. Low-skill managers, for whom the marginal benefit of obfuscation is small, optimally choose transparency and remain in the unique reputation-driven equilibrium, avoiding fragility but at the cost of poor trading performance. While opacity is commonly viewed as a driver of financial instability, our model delivers a sharply different message: the most opaque funds may not be a primary source of fragility; rather, it is funds with *intermediate* degrees of obfuscation, operated by managers of intermediate skill, that contribute to systemic fragility. This result is broadly consistent with the observation in Zuckerman (2019) that certain highly opaque or obfuscated funds maintained stable, profitable operations during the 2008 financial crisis, while many other funds experienced severe instability.

This non-monotone relationship also carries a direct policy implication. A regulatory mandate that raises the cost of obfuscation, for example, through stricter disclosure requirements, shifts the optimal-obfuscation schedule downward. Managers who previously operated in the unique low-β region may find it no longer worth incurring the higher cost of obfuscation, and optimally reduce their ω_η^* below ω_H, entering the fragile multiple-equilibria region. In other words, a transparency mandate can inadvertently push skilled, performance-driven managers into the very region of fragility it aims to suppress. Transparency thus can backfire not only by lowering fund performance and investor welfare (Subsection 3.7.2) but also by inducing financial fragility.

4.2.2 Discontinuity and Constant Obfuscation

A salient feature of Figure 8 is that ω_η^* does not increase smoothly in ω_s, but instead exhibits two upward jumps, along with a region of a constant obfuscation level between them. The jumps arise because the manager's utility function is discontinuous at the equilibrium thresholds ω_L and ω_H. At $\omega_\eta = \omega_L$, the equilibrium transitions from unique high-β to multiple equilibria: the profitable low-β equilibrium may now be realized with probability ρ_L, generating a discrete upward jump in the manager's expected utility. The same logic applies at ω_H: entering the unique low-β region eliminates the possibility of inefficient high-β trading. Because the utility gain from crossing each threshold is discrete while the obfuscation cost is smooth, optimal obfuscation responds to marginal increases in ω_s with a discrete upward jump.

The numerical results further reveal a region in which ω_η^* is pinned at ω_H and is unresponsive to the expected skill. This is due to a trade-off that arises at the boundary of the unique low-β region: a manager in the multiple-equilibria region can capture a discrete utility gain by pushing her obfuscation level exactly to ω_H, eliminating the possibility of high-β trading. Once she obfuscates at ω_H, however, the associated marginal benefit of additional obfuscation may not outweigh its marginal cost $C'(\omega_\eta)$. For managers whose skill is not high enough to justify this incremental cost, the optimal obfuscation level is pinned at ω_H. As skill rises sufficiently, the marginal benefit of obfuscation eventually dominates, and ω_η^* begins increasing again above ω_H.

These features carry concrete implications for the testable relationship between skill and obfuscation. First, while obfuscation and skill are positively correlated in general, the cross-sectional distribution of fund obfuscation should exhibit discrete mass points (i.e., clusters of funds) around the thresholds for multiple equilibria, rather than following a smooth unimodal distribution. Second, the constant-obfuscation region implies that skill is uninformative about the degree of obfuscation for a non-trivial range of managers: funds with meaningfully different skill levels optimally choose identical obfuscation levels. Within this region, however, fund performance should still be positively correlated with skill even after controlling for obfuscation levels, providing a testable prediction that disentangles the effects of skill and obfuscation.

5 Conclusion

This paper studies a model of delegated asset management in which a fund manager, privately informed about her skill, trades a risky asset while strategically designing the readability of fund communications. The model yields two stable, self-fulfilling equilibria: a Kyle-like equilibrium, in which the manager pursues profit-maximizing strategies, and a reputation-driven equilibrium, in which she trades more aggressively to signal skill, resulting in higher price volatility and lower trading profits. A mere shift in investor beliefs, unaccompanied by any change in fundamentals, can trigger a transition between equilibria, which we interpret as financial *fragility*.

We endogenize fund obfuscation and show that it serves as a commitment device: by obfuscating fund information, the manager curbs her own future showing-off incentive and steers herself toward profit-driven trading. Since this commitment value is greater for higher-skilled managers, optimal obfuscation increases with skill. This generates a non-monotone obfuscation–fragility relationship: intermediate-skill managers choose intermediate obfuscation levels, placing them in the fragile multiple-equilibria region, while high-skill managers operate stably under high obfuscation. A direct policy implication follows: transparency mandates can inadvertently push skilled managers into the fragile region and, by constraining obfuscation, induce excessive risk-taking by managers, ultimately reducing fund performance and investor welfare. In these senses, our model shows that transparency can backfire.

An interesting direction for future research would be to examine how competition among fund managers for overlapping investor bases affects obfuscation incentives, potentially generating strategic complementarities in obfuscation activities with implications for market-wide fragility.

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Online Appendix

A Proof of Lemma 3.1

From (3.1), f is observationally equivalent to $\frac{f}{\beta} = s + \frac{\eta}{\beta}$. The filtering rules of normal random variables yield results in equations (3.9)–(3.11).

B Proof of Proposition 3.1 and Corollary 3.1

By using equations (3.3) and (3.10), function F in (3.14) is represented as

$$F(\beta) = \frac{1}{2\beta\omega_s} \frac{(\beta^2\omega_s + \omega_u)(\beta^2\omega_s + \omega_\eta)^2}{(\beta^2\omega_s + \omega_\eta)^2 - \theta\omega_u\beta^2\omega_s}.$$

Denote $a = \frac{\omega_s}{\omega_\eta}\beta^2$ and $h = \frac{\omega_u}{\omega_\eta}$ so that the fixed-point problem is rewritten as

$$a = W(\alpha) \equiv h\left(1 + 2\theta\left(\frac{a}{a+1}\right)^2\right). \tag{B.1}$$

Note that $W'(a) = 4h\theta\frac{a}{(a+1)^3}$ and $W''(a) = 4h\theta\frac{1-a}{(a+1)^4}$. Also, $W'(0) = 0$ and $\lim_{a\to\infty} W'(a) = 0$. Thus, $W(a)$ is monotonically increasing in a, is convex in $a < 1$, and is concave in $a > 1$.

To find the solutions, rewrite (B.1) to obtain

$$\begin{aligned} 0 &= G(a) \\ &= a^3 + (2 - 2\theta h - h)a^2 + (1 - 2h)a - h. \end{aligned} \tag{B.2}$$

As $G(0) = -h < 0$ and $\lim_{a\to\infty} G(a) = \infty$, it has at least one solution. Also,

$$g(a) \equiv G'(a) = 3a^2 + 2(2 - 2\theta h - h)a + 1 - 2h.$$

If $2h > 1$, then $g(0) < 0$, and thus $g(a) = 0$ has a unique solution. $G(a)$ takes a U-shaped curve, and therefore, there is a unique solution to $G(a) = 0$.

Focus on $2h < 1$. Then, the discriminant of g is

$$D_g = 4\left[4\theta^2h^2 - 4h(2-h)\theta + (1+h)^2\right].$$

Thus, $g(a) = 0$ has two solutions in $a \geq 0$ if the following conditions are satisfied:

$$4\theta^2h^2 - 4h(2-h)\theta + (1+h)^2 > 0 \tag{B.3}$$

and

$$2 - 2\theta h - h < 0. \tag{B.4}$$

If either one of them is violated, $g(a) > 0$ holds for all $a \geq 0$, and $G(a)$ is monotonically increasing in a so that $G(a) = 0$ has a unique solution.

Suppose that conditions (B.3) and (B.4) are satisfied and denote the two solutions of $g(a) = 0$ as a_1 and $a_2 (> a_1)$. The local maximum and minimum of G are, respectively $G(a_1)$ and $G(a_2)$. There are three cases:

1. If, and only if, $G(a_1) > 0$ and $G(a_2) < 0$, $G(a) = 0$ has three solutions. We denote them as a_L, a_M, and a_H $(a_L < a_M < a_H)$;
2. If $G(a_1) > 0$ and $G(a_2) > 0$, then $G(a) = 0$ has a unique solution that corresponds to a_L; and
3. If $G(a_1) < 0$ and $G(a_2) < 0$, then $G(a) = 0$ has a unique solution that corresponds to a_H.

Applying $g(a_k) = 0$ $(k = 1, 2)$, it holds that

$$G(a_k) = 2a_k \left[3(1-2h) - (2 - 2\theta h - h)^2\right] \\ - (2 - 2\theta h - h)(1 - 2h) - 9h.$$

Hence, $G(a_1) > 0$ and $G(a_2) < 0$ are equivalent to the first and the second inequalities in the following:

$$a_1 < \hat{a} \equiv \frac{\theta h(1-2h) - (1+h)^2}{(1+h)^2 - 4\theta h\,(2 - h - \theta h)} < a_2.$$

These conditions are satisfied if, and only if, $g(\hat{a}) < 0$. This value is expressed as

$$g(\hat{a}) = \bar{g}R,$$

where $\bar{g} > 0$ is a function of θ and h but always positive, and

$$R \equiv 2h^3(1+2\theta)^2 + 2h^2(3 - 10\theta) + h(6 - \theta) + 2.$$

Note that $g(\hat{a}) < 0$ is a stronger condition than (B.3) and (B.4) because, if $g(\hat{a}) < 0$ with $\hat{a} > 0$, then $g = 0$ has two solutions. In summary, $G = 0$ has three solutions if, and only if, $R < 0$. By rewriting R as a function of θ, we obtain

$$R(\theta) = 8h^3\theta^2 + \theta h(8h^2 - 20h - 1) + 2(1+h)^3. \tag{B.5}$$

Taking R as a quadratic function of θ, its discriminant is $D_R = -h^2(8h-1)^3$. If $8h > 1$, then $D_R < 0$ holds, meaning that $R > 0$ for all $h > 0$. Hence, $G(a)$ is monotonically increasing in a and the solution is unique. If, in contrast, $8h < 1$, then $D_R > 0$. Also, the second term of R is negative, $8h^2 - 20h - 1 < 0$ (due to $8h < 1$). Thus, $R(\theta) = 0$ has two solutions, denoted as θ_1 and θ_2 $(> \theta_1)$, such that, $R < 0$ if, and only if, $\theta \in [\theta_1, \theta_2]$. Hence, if θ lies strictly between θ_1 and θ_2, then $G(a) = 0$ has three solutions. If $\theta \leq \theta_1$, then $G(a_1) \geq 0$ and $G(a_2) > 0$, so that case 2 arises, and the solution is unique at a_L. If $\theta \geq \theta_H$, then $G(a_1) < 0$ and $G(a_2) \leq 0$, so that case 3 arises, and the solution is unique at a_H. In summary, we obtain the following lemma:

Lemma B.1. *(i) If $8h \geq 1$, then the fixed-point problem (B.1) has a unique (stable) solution.*
(ii) If $8h < 1$, then there exist thresholds of θ denoted as θ_1 and θ_2 $(> \theta_1)$ and obtained as solutions to $R(\theta) = 0$ in (B.5). In this case, (ii-a) if $\theta \in (\theta_1, \theta_2)$, the fixed-point problem has three solutions, a_L, a_M, and a_H $(a_L < a_M < a_H)$, where a_L and a_H are stable; (ii-b) if $\theta \leq \theta_1$, then the problem has a unique solution at a_L; and (ii-c) if $\theta \geq \theta_2$, then the problem has a unique solution at a_H.

Classification of an unique equilibrium. As the definition of the high- and the low-β equilibria is not trivial in case (i) of Lemma B.1, we consider the following classification. First, note that the fixed-point problem (B.1) has the following properties:

Lemma B.2. *(i) If $\theta < \tilde{\theta} \equiv \frac{27}{16h}$, then W is less steep than the 45-degree line for all $a \geq 0$.*
(ii) If $\theta \geq \tilde{\theta}$, then there exist $a = \alpha_1$ and α_2 $(> \alpha_1)$, such that $W'(a) > 1 \Leftrightarrow a \in (\alpha_1, \alpha_2)$.

Accordingly, define $a = \hat{\alpha}$ as the smallest valeu of a such that $W'(a) = 1$. Lemma B.2 implies that

$$\hat{\alpha} = \begin{cases} \infty & \text{if } \theta < \tilde{\theta}, \\ \alpha_1 & \text{if } \theta \geq \tilde{\theta}. \end{cases}$$

We classify the high- and low-β equilibrium (i.e., a_L and a_H) in case (i) of Lemma B.1 according to the following definition.

Definition 2. *When $8h > 1$ and the solution to the fixed-point problem (B.1) is uniqe at $a = a^*$, we define the solution as a_L if $a^* < \hat{\alpha}$. Otherwise, it is classified as a_H.*

Following Definition 2, $8h > 1$ and $\theta < \tilde{\theta}$ always lead to a unique low-β equilibrium. In contrast, if $8h > 1$ and $\theta \geq \tilde{\theta}$, then we have the following result:

Lemma B.3. *If $8h > 1$ and $\theta \geq \tilde{\theta}$, the unique equilibrium of the fixed-point problem (B.1) is always the high-β equilibrium.*

Proof. Denote the solution to $a = W(a)$ as a^*. It suffices to show that $\theta \geq \tilde{\theta} \Rightarrow a^* > 1$, as $a = 1$ is the tipping point of the hump-shaped curve $W'(a)$ and $\alpha_1 < 1$. Note that $a^* > 1$ is equivalent to $W(1) > 1$, which in turn reduces to $\theta > \hat{\theta} \equiv \frac{9}{2}(\frac{1}{2h} - 1)$. Since $8h > 1 \Leftrightarrow \tilde{\theta} > \hat{\theta}$, $\theta \geq \tilde{\theta}$ implies $a^* > 1 > \alpha_1$. □

Characterizing the boundaries by ω_η. Suppose that $8h < 1$. Note that

$$h\frac{\partial R}{\partial h} = 2\theta h(1 + 10h) - 6(1 + h)^2.$$

Denote

$$\bar{\theta} \equiv \frac{3(1 + h)^2}{h(1 + 10h)},$$

so that $\frac{\partial R}{\partial h} \gtreqless 0 \Leftrightarrow \theta \gtreqless \bar{\theta}$. At $\theta = \bar{\theta}$, $R(\bar{\theta}) < 0$ due to $8h < 1$. Therefore, $\theta_1 < \bar{\theta} < \theta_2$ hold. Also,

$$\frac{\partial R(\theta_1)}{\partial h} < 0, \; \frac{\partial R(\theta_2)}{\partial h} > 0.$$

Since $\frac{\partial R(\theta_1)}{\partial \theta} < 0$ and $\frac{\partial R(\theta_2)}{\partial \theta} < 0$, the implicit function theorem implies

$$\frac{d\theta_k}{dh} = -\frac{\partial R(\theta_k)/\partial h}{\partial R(\theta_k)/\partial \theta} < 0$$

for both $k = 1$ and 2. Thus, θ_1 and θ_2 are monotonically increasing in ω_η. Also, at the limit of $h \to \frac{1}{8}$, it holds that $\theta_1 \to \theta_2$. Hence, by the squeeze theorem, $\theta_1 = \theta_2 = \frac{3(1+h)^2}{h(1+10h)}|_{h=\frac{1}{8}} = \frac{27}{2}$ at this limit. Moreover, $\tilde{\theta} = \frac{27}{16h}$ is increasing in ω_η with $\tilde{\theta} = 0$ at $\omega_\eta = 0$ and $\tilde{\theta} = \frac{27}{2}$ at $\omega_\eta = 8\omega_u$. Therefore, we define a unique ω_η that satisfies $\theta_1(\omega_\eta) = \theta$ as ω_H and that satisfies $\theta_2(\omega_\eta) = \theta$ as ω_L. Finally, ω_0 is defined by $\tilde{\theta}(\omega_\eta) = \theta$. Due to the monotone mapping, ω_0, ω_L, and ω_H are increasing in θ.

C Proof of Corollaries 3.3 and 3.4

Note that the solutions to the fixed-point problem (B.1) lies in $a > h$, suggesting that $\beta > \sqrt{\frac{\omega_u}{\omega_s}}$ always holds. The unconditional expected performance is given by (3.16), and $\mathrm{E}[\pi]$ is monotonically decreasing in β. Hence, Corollary 3.4 directly follows from Corollary 3.3.

To show Corollary 3.3, rewrite the fixed-point problem in (B.1) as $0 = J(z, \omega_\eta)$ where

$$J(z, \omega_\eta) = (z - 1)\left(z + \frac{\omega_\eta}{\omega_u}\right)^2 - 2\theta z^2, \tag{C.1}$$

with $z \equiv \beta^2 \frac{\omega_s}{\omega_u}$. Also, denote $z_k = \beta_k^2 \frac{\omega_s}{\omega_u}$ for $k = L, H$. Since $\frac{\partial J(z_k,\omega_\eta)}{\partial \omega_\eta} > 0$ and $\frac{\partial J(z_k,\omega_\eta)}{\partial \beta} > 0$, the implicit function theorem implies $\frac{d\beta_k}{\omega_\eta} < 0$ for $k = L, H$.

D Proof of Proposition 4.1

In this proof, we show (i) the positivity of ω_η^*, (ii) that ω_η^* is non-decrease in ω_s, and (iii) ω_η^* strictly increases with ω_s when interior to a regime.

Part (i): $\omega_\eta^* > 0$ for all $\omega_s > 0$. Define the manager's gross fee revenue (benefit) as

$$B(\omega_\eta) \equiv \phi(1+\theta)\sqrt{\omega_u\omega_s}\mathrm{E}\left[\frac{\sqrt{z}}{z+1}\right],$$

where z is a solution to the fixed-point problem in (C.1). Note that $U(\omega_\eta) = B(\omega_\eta) - C(\omega_\eta)$. We show that the right-derivative of U at $\omega_\eta = 0$ is strictly positive, so $\omega_\eta = 0$ cannot be optimal.

At $\omega_\eta = 0$, the fixed-point problem lies in the unique-z_H regime (since $\omega_\eta < \omega_L$ for any $\omega_L > 0$). In this regime $\mathrm{E}[\sqrt{z}/(z+1)] = \sqrt{z_H}/(z_H+1)$, and the marginal benefit is

$$\left.\frac{dB}{d\omega_\eta}\right|_{\omega_\eta=0^+} = \phi(1+\theta)\sqrt{\omega_u\omega_s} \cdot \left.\frac{d}{dz}\left(\frac{\sqrt{z}}{z+1}\right)\right|_{z=z_H} \cdot \left.\frac{dz_H}{d\omega_\eta}\right|_{\omega_\eta=0^+}.$$

We have $\frac{d}{dz}(\sqrt{z}/(z+1)) < 0$ and $dz_H/d\omega_\eta < 0$, so their product is strictly positive. Since $\omega_s > 0$, the right-derivative of B at zero is strictly positive. Since $C'(0) = 0$,

$$\left.\frac{dU}{d\omega_\eta}\right|_{\omega_\eta=0^+} = \left.\frac{dB}{d\omega_\eta}\right|_{\omega_\eta=0^+} - C'(0) > 0,$$

so U is strictly increasing at $\omega_\eta = 0$, which implies $\omega_\eta^* > 0$. Since C' is assumed sufficiently large for large ω_η (ensuring SOC and that $U \to -\infty$), a finite interior or boundary maximizer exists with $\omega_\eta^* > 0$.

Part (ii): ω_η^* is non-decreasing in ω_s. Write $U(\omega_\eta; \omega_s) = \phi(1+\theta)\sqrt{\omega_u \omega_s} \cdot Q(\omega_\eta) - C(\omega_\eta)$, where

$$Q(\omega_\eta) \equiv \mathrm{E}\left[\frac{\sqrt{z}}{z+1}\right]$$

is the expected value of $\sqrt{z}/(z+1)$ under the equilibrium selection rule. Note that Q depends on ω_η but not on ω_s, since ω_s enters U only through the scalar coefficient $\phi(1+\theta)\sqrt{\omega_u\omega_s}$.

Observe that $U(\omega_\eta; \omega_s)$ satisfies strictly increasing differences in (ω_η, ω_s): for any $\omega_\eta'' > \omega_\eta'$ and $\omega_s'' > \omega_s'$,

$$\begin{aligned} &U(\omega_\eta''; \omega_s'') - U(\omega_\eta'; \omega_s'') - \left[U(\omega_\eta''; \omega_s') - U(\omega_\eta'; \omega_s')\right] \\ &\quad = \phi(1+\theta)\sqrt{\omega_u}\left(\sqrt{\omega_s''} - \sqrt{\omega_s'}\right)\left[Q(\omega_\eta'') - Q(\omega_\eta')\right]. \end{aligned}$$

This expression is strictly positive if and only if $Q(\omega_\eta'') > Q(\omega_\eta')$, i.e., if Q is increasing in ω_η. We now verify this. Consider each regime separately. Under unique-z_H and z_L regimes, the claim holds due to $\frac{d}{dz}(\sqrt{z}/(z+1)) < 0$ and $dz_k/d\omega_\eta < 0$ for $k = L, H$. Also, under multiple-equilibria regime, the same claim holds as Q becomes the weighted average of $\sqrt{z_k}/(1+z_k)$. Now, consider the boundaries: As ω_η crosses ω_L upward, the realized equilibrium shifts from z_H alone to the mixture $(\rho, z_L; 1-\rho, z_H)$. Since $z_L < z_H$ and $\sqrt{z}/(z+1)$ is decreasing in z, we have $\sqrt{z_L}/(z_L+1) > \sqrt{z_H}/(z_H+1)$, so introducing z_L with positive weight $\rho > 0$ causes a discrete upward jump in Q. Similarly, as ω_η crosses ω_H upward, the mixture shifts to z_L alone, causing another discrete upward jump in Q (since $z_L < z_H$ implies $\sqrt{z_L}/(z_L+1) > (1-\rho)\sqrt{z_H}/(z_H+1) + \rho\sqrt{z_L}/(z_L+1)$ for $\rho < 1$). Therefore Q is strictly increasing in ω_η everywhere: continuously within each regime, and with upward jumps at the boundaries.

Returning to the main argument: the cross-difference is strictly positive for all $\omega_\eta'' > \omega_\eta'$ and $\omega_s'' > \omega_s'$, so U has strictly increasing differences in (ω_η, ω_s). By the monotone comparative statics theorem (Topkis, 1998), every selection from the argmax correspondence $\omega_\eta^*(\omega_s)$ is non-decreasing in ω_s.

Part (iii): ω_η^* is strictly increasing in ω_s when interior to a regime. Suppose $\omega_\eta^*(\omega_s)$ is interior to one of the three regimes, that is, $\omega_\eta^*(\omega_s) \notin \{0, \omega_L, \omega_H\}$ and ω_η^* does not lie at a boundary of its regime. Then ω_η^* satisfies the interior first-order condition

$$\phi(1+\theta)\sqrt{\omega_u\omega_s}\cdot Q'(\omega_\eta^*) = C'(\omega_\eta^*), \tag{FOC}$$

where $Q'(\omega_\eta) \equiv dQ/d\omega_\eta > 0$ within each regime. Define

$$H(\omega_\eta, \omega_s) \equiv \phi(1+\theta)\sqrt{\omega_u\omega_s}\cdot Q'(\omega_\eta) - C'(\omega_\eta).$$

By the implicit function theorem, since the second-order condition holds (i.e., $\partial H/\partial\omega_\eta < 0$ at the optimum),

$$\frac{d\omega_\eta^*}{d\omega_s} = -\frac{\partial H/\partial\omega_s}{\partial H/\partial\omega_\eta}.$$

We compute

$$\frac{\partial H}{\partial\omega_s} = \phi(1+\theta)\sqrt{\omega_u}\cdot\frac{1}{2\sqrt{\omega_s}}\cdot Q'(\omega_\eta^*) > 0,$$

since $Q'(\omega_\eta^*) > 0$ within each regime. By the second-order condition, $\partial H/\partial\omega_\eta < 0$. Therefore

$$\frac{d\omega_\eta^*}{d\omega_s} = -\frac{\partial H/\partial\omega_s}{\partial H/\partial\omega_\eta} > 0,$$

so ω_η^* is strictly increasing in ω_s at any interior point of a regime.