FINA 3103: Intermediate Investment
Class Note: Fixed Income Securities (cont'd)

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What we have derived so far

1. Yield to maturity, y:

$$P = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t} + \frac{FV}{(1+y)^T} \Rightarrow y$$

2. T-year spot rate, $r_{0,T}$:

$$P_{0,T} = \frac{FV}{(1+r_{0,T})^T} \Rightarrow 1 + r_{0,T} = ((1+r_1)\cdots(1+r_T))^{1/T}$$

3. Forward rate, f_t:

$$1 + f_t = \frac{P_{0,t-1}}{P_{0,t}} \overset{\text{implies}}{=} 1 + r_t$$

- Q: f_t is derived from P and indicative of r_t. How they are related?
    - Consider uncertainty in bond holding

Risk in Fixed-income Securities

Expectation Hypothesis

How to interpret f_t? How investors form belief about f_t?

Expectations Hypothesis

Forward rates predict expected future (one-year) spot rates

$$f_t = \mathbb{E}[r_t]$$

- f_t = market's expectations of future short-term interest rates
  - Shape of the yield curve/term structure changes depending on investor expectation
- But what types of risks does the market incorporate? Can we separate them?

Liquidity Risk

Liquidity Premium

Forward rate reflects liquidity premium (or term premium):

$$f_t = \mathbb{E}[r_t] + \underbrace{LP}_{\text{liq. premium}}$$

- Bonds are traded in (illiquid) OTC
  - Search cost to find trading counterparts
- The longer the maturity, the more illiquid the market is
  - Clientele and interest rate risk
- LP = compensate for liquidity risk

Example: Liquidity Premium

Example

You want to invest in ZCB with $FV = \$1,000$ for one year but are torn between (1) buying one year ZCB at $P_{0,1} = \$952.4$ and (2) buying 2 year ZCB today at $P_{0,2} = \$907.03$ and sell it a year later. Initially, there is no liquidity risk.

- Compute $f_2 = P_{0,1}/P_{0,2} - 1 = 0.05$ (exp. hypothesis level)
- The price of 2-year ZCB one year later:

$$\mathbb{E}[P_{0,1}] = \frac{\$1,000}{1+f_2} = \frac{\$1,000}{1.05} \approx \$952.4$$

- Return from (2) is $(\mathbb{E}[P_{0,1}] - P_{0,2})/P_{0,2} = \frac{952.4}{907.03} - 1 \approx 5\%$
- So, (1) = (2) if <u>no liquidity premium</u>
- What if investors require higher return due to liquidity risk?

Example: Liquidity Premium

Example

Investors are now concerned about liquidity risk of 2y ZCB and demand a higher discount. As a result, the bond price is $P_{0,2} = \$890$. What is the liquidity premium?

Default Risk

Chinese corporate bond defaults likely to continue to increase in 2022, Moody's says

- Offshore corporate bond defaults rose 28 per cent in first three quarters of 2021
- Property developers, such as China Evergrande Group and Kaisa Group, have faced liquidity crunches in recent months

Chad Bray + FOLLOW
Published: 4:01pm, 17 Nov, 2021

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212

Post

An Evergrande housing complex in Beijing. The company, China's biggest developer, has bought itself more time by making last minute payments after a series of asset sales. Photo: AFP

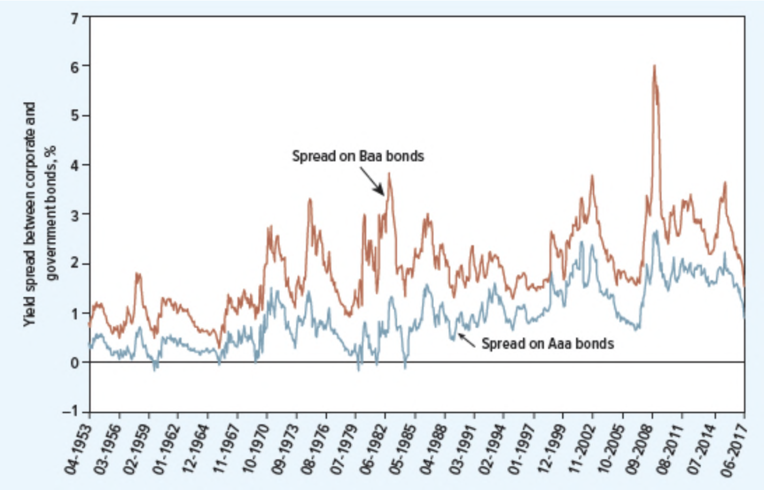
Default Risk

Non-gov. bonds carry default risk

- Default: a debt issuer fails to make a promised payment (C or FV)
- Rating agencies (e.g., Moody's and S&P) provide credit rating
  - Likelihood of default by each issuer

Credit Risk	Moody's	S&P	Fitch
Investment Grade			
Highest Quality	Aaa	AAA	AAA
High Quality (Very Strong)	Aa	AA	AA
Upper Medium Grade (Strong)	A	A	A
Medium Grade	Baa	BBB	BBB
Not Investment Grade			
Somewhat Speculative	Ba	BB	BB
Speculative	B	B	B
Highly Speculative	Caa	CCC	CCC
Most Speculative	Ca	CC	CC
Imminent Default	C	C	C
Default	C	D	D

Default Risk



Bond yields reflect default risks?

Yield rate is decomposed into several parts:

- Promised YTM: yield if default does not occur
- Expected YTM: yield of the probability-weighted average of all possible cash flows
  - Expectation incorporating default risk
- Default premium: difference between promised yield and expected yield
- Risk premium (of a bond): difference between the expected yield on a risky bond and the yield on a risk-free bond of similar maturity and coupon rate

Decomposition of Corporate Bond Yields

Example

10-year Treasury ZCB (no risk) with $FV = \$1k$ is traded at \$463.19, yielding 8%. In contrast, 10-year ZCB with $FV = \$1k$ issued by firm A is traded at \$321.97. Expected redemption of A's ZCB is \$762.22.

- Promised y:

$$\$321.97 = \frac{\$1,000}{(1 + y_{Prom})^{10}} \Rightarrow y_{Prom} = 12\%$$

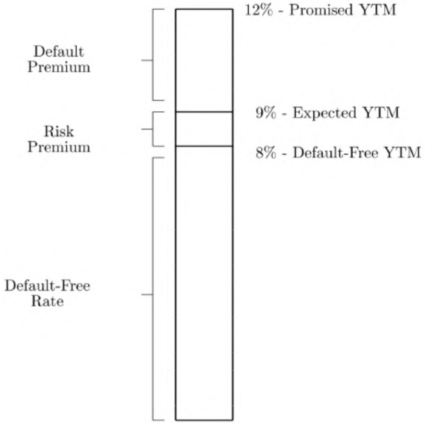
- Expected y:

$$\$321.97 = \frac{\$762.22}{(1 + y_{Exp})^{10}} \Rightarrow y_{Exp} = 9\%$$

- Default premium is $12\% - 9\% = 3\%$
- Risk premium is $9\% - 8\% = 1\%$

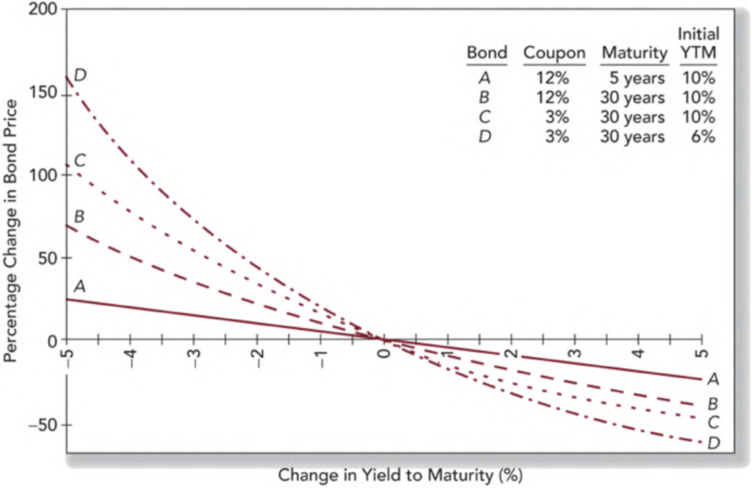
Decomposition of Corporate Bond Yields

- Default premium: compensation for expected loss
- Risk premium: compensation for taking such risks



Interest Rate Risk

- Interest rate has substantial impact on bond value: $P = \sum \frac{CF_t}{(1+y)^t}$



Duration and Interest Rate Risk

What happens to the bond price if the interest rate changes?

- Denoting cash flow as CF_t

$$P = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t}$$

- Consider how $y \to y + \Delta y$ leads to $P \to P + \Delta P$
- With small $\Delta y = dy$, taking derivative is good approximation:

$$\frac{\Delta P}{\Delta y} = \frac{dP}{dy} = -\sum_{t=1}^{T} \frac{CF_t}{(1+y)^{t+1}} t$$

  - In general, negative impact

Duration and Interest Rate Risk

- Consider % change in price:

$$\frac{\Delta P}{\Delta y} = -\sum_{t=1}^{T} \frac{CF_t}{(1+y)^{t+1}} t$$

$$\Rightarrow \underbrace{\frac{\Delta P}{P}}_{\text{\% change in } P} = \underbrace{-\frac{1}{P} \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} t}_{\text{"impact coefficient"}} \times \underbrace{\frac{\Delta(1+y)}{1+y}}_{\text{\% change in } y} \qquad (1)$$

Duration and Interest Rate Risk

Duration

Duration is (i) a measure of a bond's sensitivity to changes in interest rates and (ii) the weighted average time to earn all cash flow.

- Macaulay Duration D is the (absolute) sensitivity.
- From (1):

$$\frac{\Delta P}{P} = -\frac{1}{P}\sum_{t=1}^{T}\frac{CF_t}{(1+y)^t}t\frac{\Delta(1+y)}{1+y}$$

$$\therefore D \equiv \frac{1}{P}\sum_{t=1}^{T}\frac{CF_t}{(1+y)^t}t$$

- D: impact of changes in y on P (in %)
- Captures the risk of interest rate changes (e.g., MP shocks)

Modified Duration

- Define "modified" duration as

$$D_{mod} = \frac{D}{1+y}$$

- Then

$$\frac{\Delta P}{P} = -D\frac{\Delta(1+y)}{1+y} = -D_{mod}\Delta y$$

- D^* is the impact of *absolute change* (not %) in y on % change in P

Duration and Interest Rate Risk

Example

Consider a bond with duration of $D =$1.8852 (years). When the annual yield changes from 5% to 6%, how much will the bond price change?

- Compute modified duration:

$$D^* = \frac{D}{1+y} = \frac{1.8852}{1.05} \approx 1.795$$

- Then, compute the impact on the price

$$\Delta P/P = -D^* \Delta y = -1.795 \times \underbrace{0.01}_{0.06-0.05} = -0.01795$$

- But why duration is expressed in time?

Risk and Duration

- Why is D also related to (defined in) time periods?
- Holding bonds = fixed contract to lend money to the government
  - What if the interest rate changes?
  - How does the contract period affects the risk?

Interest rate risk

You bought T-year coupon bond with $FV = \$100$ and $C = 1\%$ at price $P = \$100$ (so $y = 1\%$). Just after you bought it, new bond is issued at the same price but $C = 2\%$, meaning $y = 2\%$ return. How does the price of your old bond change? Compare the impact when $T = 1$ and $T = 10$.

Risk and Duration

Non-arbitrage = old bond must yield same return as new one

- $T = 1$:

$$P' = \frac{101}{1 + 0.02} \approx 99.02$$

  - To yield 2% return, it must be available at $P' = \$99.02$

- $T = 10$:

$$P' = \sum_{t=1}^{10} \frac{1}{(1.02)^t} + \frac{101}{(1.02)^{10}} \approx 90$$

  - To yield 2% return <u>for 10 years</u>, it must be traded at $90

- **General observation**:
  - $y \uparrow$ reduces PV of future CF
  - But <u>its impact is larger for CFs that are further in the future</u>
  - Long maturity $\rightarrow$ large exposure to interest rate risk

Implications of Duration

Duration is weighted average of time

$$D \equiv \sum_{t=1}^{T} \frac{1}{P} \frac{CF_t}{(1+y)^t}\, t$$

$$= 1 \times \frac{\text{PV of CF}_1}{P} + 2 \times \frac{\text{PV of CF}_2}{P} + \cdots + T \times \frac{\text{PV of CF}_T}{P}$$

- The effective maturity of a bond incorporating the CF

Implications of Duration

Duration and Effective Maturity

Consider a bond with $T = 5$, $FV = \$1,000$, and $C = 4\%$. Current market YTM is $y = 4.5\%$.

Time	CF	PV of CF	PV of t weighted CF	Weight on t
1	40	38.28	38.28	0.0391
2	40	36.63	73.26	0.0374
3	40	35.05	105.16	0.0358
4	40	33.54	134.17	0.0343
5	1,040	834.55	4172.75	0.8533
total		$P = 978.05$	4,523.61	1.0

- $D = \sum t \times \text{weight}_t = 4.63$ years

Implications of Duration

- How long it takes on average to receive all CF?
  - Ignore y and let's say you receive 20% of CF each year for next 5 years
  - The average time to get all CF is

$$D = \sum_{t=1}^{5} \frac{1}{5} t = \frac{15}{5} = 3\text{years}$$

- In general, $D \leq T$ ($D = T$ only for ZCB)
- Long D = longer time to receive all CFs
  = exposed to the contract for longer periods

Implications of Duration (cont'd)

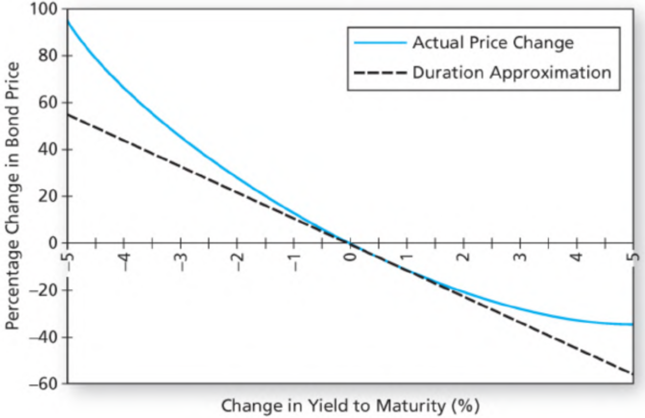
- Holding T constant, D is longer when C is lower
  - Why?
- Holding C constant, D increases with T
  - Why?

Duration and Price Sensitivity

- We derived

$$\Delta P/P = -D^{*}\Delta y$$

- In reality, it looks convex



Convexity

- The relationship between bond prices and yields is not linear
- Duration rule is a good approximation for only small changes in bond yields
  - $dy \approx \Delta y$ is true only if Δy is very small
- For larger changes, we need to correct for convexity

Convexity Correction

- Define convexity as

$$\text{Convexity} = \frac{1}{P(1+y)^2} \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} t(t+1)$$

- The corrected sensitivity equation is given by

$$\frac{\Delta P}{P} = -D^* \Delta y + \frac{(\Delta y)^2}{2} \times \text{Convexity}$$

- No need to memorize the equation
  - the point is the duration is not a perfect measure
  - for a sizable change in y, we need a correction (convexity)

Bond Risk Management

Portfolio Duration

- You cannot control the duration of the individual fixed-income security, D_i
- But if you hold bonds with different duration $(D_1, \cdots, D_N)$, your portfolio duration becomes

$$D_p = \sum_{i=1}^{N} w_i D_i$$

- So you can control the duration of your position by holding a portfolio

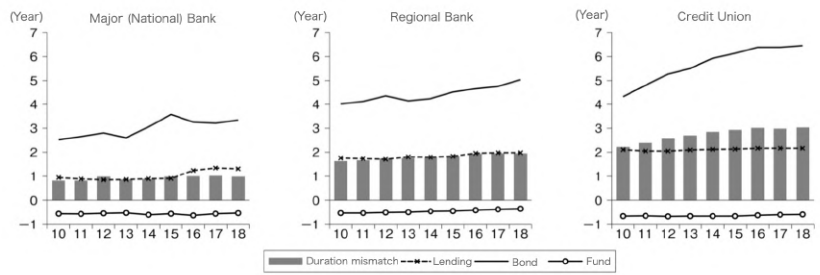
Bond Portfolio Strategies

- **Passive management**: replicate some large index (indexing)
- **Immunization strategies**
  - By controlling portfolio's D_p, offset interest-rate risk
  - Establish a virtually zero-risk bond portfolio, interest rate fluctuations have no impact
  - Sometimes called *quasi* passive strategy
- **Active management**: try to outperform bond index

Risk Management

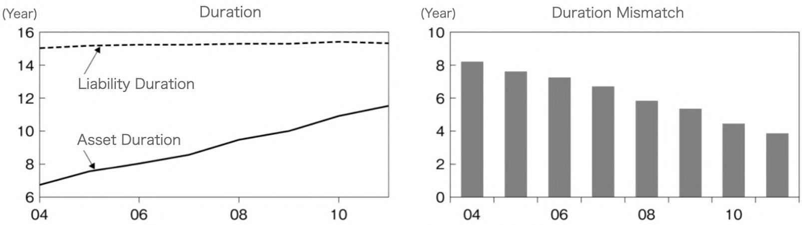
- Firms hold/issue bonds but must manage the risk
  - For business purposes
  - To comply with regulation (Basel regulation)
- Different institutions have different duration on balance sheet
  - Bank, insurance company, pension funds, central bank
- In general, try to match the duration of asset and liability
  - Called Asset Liability Management (ALM)

Risk Management: Banking Sector



- All types have duration mismatch (taking interest-rate risk)
  - What happens if $y \uparrow$?
- Basel regulation requires a certain level of risk-equity ratio

Risk Management: Life Insurance Companies



9 Largest Life Insurance Companies
(Source: Sugawa et al., 2012)

- Duration mismatch is substantial but shrinking
  - Economic Solvency Ratio ("Solvency Margin Regulation")

ALM and Immunization

- You are obligated to pay $\$X$ in T years from now
  - Or equivalently, you have some loans on your liability side
- By buying bonds, you can immunize this risk
  - Could use zero-coupon bonds and coupon bonds
- The target is to equalize duration of both sides

$$D_{asset} = D_{liability}$$

Immunization by Zero-Coupon Bonds

Example

Suppose that you work for an insurance company and issued a 5-year 8% guaranteed investment contract (GIC) for $10,000. How to immunize this payment by buying zero-coupon bonds?

- Obligated to pay $\$10,000 \times (1.08)^5 = \$14,693.28 (\equiv X)$ in 5 years
- If you buy Q units of ZCB so that

  $$Q \times FV = X$$

  then you have zero CF on target date
- No impact from interest rate

Immunization by Coupon Bonds

Example

Suppose that you work for an insurance company and issued a 5-year 8% guaranteed investment contract (GIC) for $10,000. How to immunize this payment by buying coupon bonds?

- ZCBs are not often available with intended maturities
- Coupon bonds have $D < T$
- Construct portfolio to set $D_{port} = T_{liability}$ ($= D_{liability}$ in this ex)
- Changes in y does not affect your net portfolio

Limitation

- D will be affected by various factors
  - Ex., bonds' residual maturity and interest rate
- Requires continuous rebalancing
  - Attention costs
  - Fees and commissions
  - Computation errors
- Convexity problem: duration formula works only if Δy is small

Banking Crisis

Silicon Valley Bank collapse, 2023

IDES OF MARCH: Days after Silicon Valley Bank collapsed, customers line up to try and retrieve their funds.

MARCH 8: Silicon Valley Bank (SVB) announces $1.75 billion capital fundraising.

MARCH 9: SVB stocks plummet by 60%.

MARCH 10: SVB collapses after a bank run fueled by social media. Government regulators take over the bank.

MARCH 12: U.S. government announces it will guarantee all deposits at SVB.

MARCH 13: HSBC buys U.K. arm of SVB.

MARCH 14: Moody's Investor Service adds six regional banks to the review list for credit rating downgrades.

MARCH 17: SVB Financial Group, Silicon Valley Bank's former parent company, files for Chapter 11 bankruptcy protection.

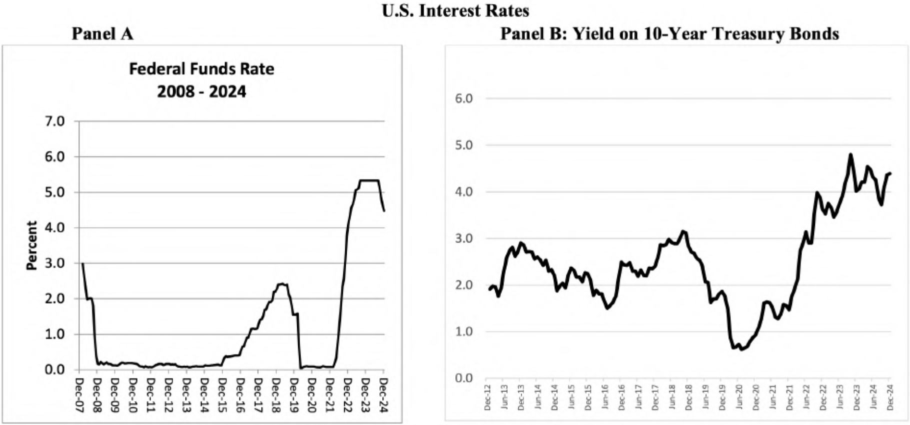
MARCH 26: First Citizens Bank buys Silicon Valley Bank.

PHOTOGRAPH: JUSTIN SULLIVAN/GETTY IMAGES

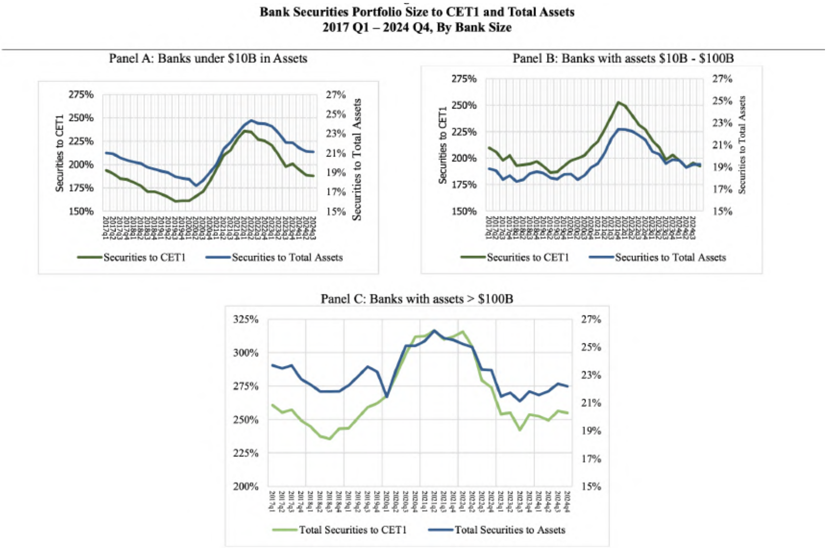
Banking Crisis

- SVB failure and subsequent banking crisis
  - Jiang et al (2024) and Cole et al (2025) document accumulated interest-rate risk
  - $r \uparrow$ in 2022-23 in the US as a trigger
- S&L crisis in 1970s–80s was also about the interest-rate risk
- Unlike the Global Financial Crisis (2008-) caused by credit risk

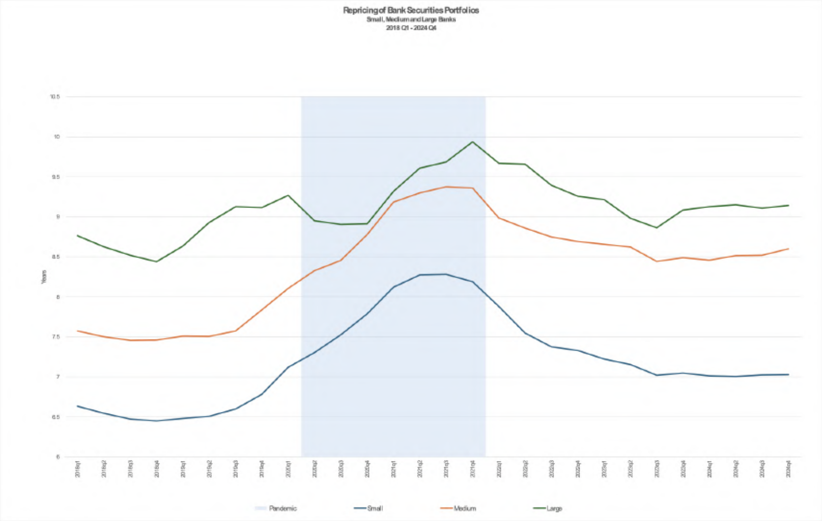
Interest Rate (Cole, et al., 2025)



Portfolio Size (Cole, et al., 2025)



Duration (Cole, et al., 2025)



What Happened?

- We did not learn from S&L crisis in 1970s?

1. Evaluating banks' interest-rate risk is inherently complex
   - Utility company: discounted CF by r
   - Bank: CF depends heavily on r and is discounted by r
2. Not hedging interest-rate risk can be optimal
   - Leverage ratio reg. (Basel III), Risk-weighted capital requirement
   - Hedging requires long-term ⇒ short-term debt on asset side
   - It limits the growth of equity and will tighten leverage constraint
   - Krishnamurthy et al., (2025) "hedging is not optimal"