When Transparency Backfires: Strategic Obfuscation and Financial Fragility*

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Abstract

We study delegated asset management in which a fund manager, informed about her skill, obfuscates fund communications to investors. Under intermediate opacity, two self-fulfilling equilibria coexist: one with performance-maximizing trading, and the other with excessive risk-taking, giving rise to financial fragility. The manager deliberately chooses opacity as a commitment device, curbing excessive risk-taking by her future self. Since skilled managers benefit more from obfuscation, optimal opacity increases with skill, generating a non-monotone opacity–fragility relationship: intermediate-opacity funds are the primary source of fragility, while highly opaque funds operate stably. Transparency mandates can push skilled managers into the fragile region, meaning transparency backfires.

JEL classification: G11, G12, G14, G23

Key words: informed trading, delegated asset management, opacity, reputation, financial fragility

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1 Introduction

Obfuscation is common in investor-directed fund communications. Even when pursuing simple strategies, fund managers describe them in markedly different and often unnecessarily complex ways. For example, as deHaan, Song, Xie, and Zhu (2021) document, two funds that both track the S&P 500 describe essentially identical objectives with strikingly different clarity: Schwab states in a single sentence, "The fund's goal is to track the total return of the S&P 500 Index," while Deutsche describes it in 60 words, embedding the same objective in technical language.[1] Similar patterns arise in fund names: labels such as "Dynamic Opportunity Fund," "Flexible Growth Strategy," or "Opportunistic Equity Fund" convey little concrete meaning to unprofessional retail investors at first glance, yet in practice often correspond to relatively simple large-cap portfolios that closely resemble the S&P 500. This pattern is puzzling, as standard signaling theory suggests that managers, seeking to convey skill and attract capital, should prefer greater transparency rather than opacity. Why would managers make information harder for their own investors to interpret, and what are the incentives and market consequences when fund opacity becomes a manager's strategic choice?

To address these questions, we study an equilibrium model of financial market trading with delegated asset management. A fund manager, privately informed about her skill, raises capital from investors through obfuscated communications and trades a risky asset in the financial market à la Kyle (1985). We define skill as the manager-specific ability to uncover more information about asset payoffs than the market: the more skilled the manager, the larger the mispricing she can identify and the greater the trading profits she can potentially earn. To attract fund flows, the manager privately communicates information about her risky position (portfolio) to investors. Investors, in

[1] Deutsche's S&P 500 index fund is described as "The fund seeks to provide investment results that, before expenses, correspond to the total return of common stocks publicly traded in the United States, as represented by the Standard & Poor's 500 Composite Stock Price Index (S&P 500 Index). The fund invests for capital appreciation, not income; any dividend and interest income is incidental to the pursuit of its objective."

turn, use this fund information to infer the manager's skill and decide on capital allocation to the fund. Importantly, the manager strategically chooses the readability of this communication: by obfuscating information, she makes it more difficult for investors to back out her underlying skill.[2]

The model yields two stable, self-fulfilling equilibria regarding the manager's trading strategy. One of the equilibria is similar to that of Kyle (1985), where the manager employs the profit-maximizing trading strategy. In the other, which we call the "flow-driven" equilibrium, the manager trades the risky asset more aggressively to appear skilled and to attract large fund flows. This strategy increases trading volume, price volatility, and market liquidity but undermines the fund's expected performance by revealing too much private information to the market maker. Intuitively, the manager can boost investors' perception of her skill by placing a large order, either buy or sell, because such an action makes her appear as a skilled manager who has identified an asset with significant mispricing. Although the expected performance deteriorates, it triggers large fund flows, and the manager earns a large fee income. Key to our multiple equilibria is the manager's conflicting motives for revealing skill: on one hand, she wishes to hide her skill from the market maker to earn trading profits, but on the other hand, she wants to show it off to investors to establish high reputation and attract large fund flows.

The relative strength of these competing motives is governed by the degree of fund opacity. When opacity is high, investors can extract little information from fund communications and do not rely on them to infer skill. Thus, flows are insensitive to the manager's trading strategy, the showing-off motive is suppressed, and only the Kyle-like equilibrium survives. When opacity is low, in contrast, investors rely heavily on fund information, and only the flow-driven equilibrium exists. At intermediate levels of opacity, however, both equilibria coexist, and which one prevails is determined by investors' beliefs. If investors expect aggressive, flow-driven trading, they treat fund information as a reli-

[2]Following Ellison and Ellison (2009), the term "obfuscation" refers to general practices that reduce the readability of information and distort communications. Accordingly, "transparency" and "opacity" refer to the degree of interpretability and precision of information with which investors can assess the fund's positions, rather than mere disclosure.

able signal of skill, making flows highly responsive to reported positions. It strengthens the showing-off motive and induces the manager to trade aggressively, confirming the investors' beliefs. The symmetric argument sustains the Kyle-like equilibrium when investors expect profit-maximizing behavior. A mere shift in beliefs, unaccompanied by any change in fundamentals, can therefore trigger a transition from the Kyle-like to the flow-driven equilibrium, where price volatility is high and trading performance is poor. We interpret this susceptibility to non-fundamental belief shifts as *fragility*.[3]

Building on these results, we endogenize fund opacity and study which types of managers contribute to financial fragility. By obfuscating fund information, a manager weakens investors' responsiveness to reported positions and curbs her showing-off incentive at the portfolio-decision stage. Although, when deciding on her risky position, she takes investors' belief-updating rule as given and is tempted to over-trade to attract flows, she can *ex-ante* influence that rule through obfuscation. Opacity thus serves as a commitment device: by tying her own hands, the manager mitigates inefficient risk-taking, raises the profit margin of her risky position, and improves the fund's expected performance, thereby benefiting both herself and investors.

The value of this commitment is greater for higher-skilled managers, as they benefit more from improved profit margins. Optimal opacity therefore increases with managers' skill. This result is consistent with the "brain drain" documented by Kostovetsky (2017), whereby top mutual fund managers migrate to more opaque hedge funds and deliver superior performance. Crucially, the relationship between opacity and fragility is non-monotone. While high opacity yields a unique Kyle-like equilibrium and low opacity yields a unique flow-driven equilibrium, it is at intermediate levels of opacity where multiple equilibria, and thus fragility, arise. Because intermediate-skilled managers optimally choose intermediate opacity, they expose financial markets to belief-driven instability. Such behavior is consistent with momentum crashes and sudden reversals observed during financial crises (Brunnermeier, 2009), as well

[3] As in Greenwood and Thesmar (2011), we define a market as fragile if it is susceptible to non-fundamental demand shifts, such as those caused by changes in investor beliefs.

as the abrupt strategy unwinding among quantitative funds documented by Khandani and Lo (2011). Importantly, fragility in our model is driven entirely by shifts in market beliefs and can arise without fundamental shocks, in line with the argument that large historical price swings are rarely accompanied by proportionally large changes in fundamentals (Gennotte and Leland, 1999). A direct policy implication follows: transparency mandates may push managers toward lower opacity and move high-skilled managers into the fragile intermediate-opacity region, thereby *increasing* rather than decreasing systemic fragility. Transparency, in this sense, backfires.

Our paper contributes to the understanding of opacity and obfuscation in financial markets. Empirically, Edelen, Evans, and Kadlecc (2012), Badoera, Costellob, and Jamesc (2020), and deHaan, Song, Xie, and Zhu (2021) document that mutual funds strategically obfuscate disclosures of their strategies and fee structures.[4] Theoretically, Carlin (2009), Carlin and Manso (2010), and Sato (2014) model complexity as a tool for extracting fees or preserving informational advantages, but treat opacity as exogenously given. By contrast, we endogenize fund managers' obfuscation choices and show that opacity mitigates managers' showing-off motive and improves both managers' and investors' *ex-ante* utility, not as a means to exploit less-informed investors.

Our paper is also related to the literature on fragility and trading frenzies in financial markets. Cespa and Vives (2015) is most closely related, as they also find multiple equilibria arising from short-term investor horizons and persistent liquidity trading, and Froot, Scharfstein, and Stein (1992) likewise emphasize strategic complementarities among short-term traders. Unlike these studies, our model does not feature short horizons; fragility instead arises from managers' concerns about fund flows. While Veldkamp (2006) and Goldstein, Ozdenoren, and Yuan (2013) generate trading frenzies through strategic complementarities in information markets or real-investment feedback, our flow-driven equilibrium arises because managers trade aggressively out of fear

[4] Célérier and Vallée (2019) show that sellers of structured financial products deliberately engineer complexity into their offerings, suggesting that strategic opacity is a common practice among sellers of financial products more broadly.

of being perceived as less skilled by investors who expect aggressive trading.[5]

Lastly, our work contributes to the broader understanding of asymmetric information about skills in financial markets. A large literature studies risk-taking when managerial ability is private but inferred through performance signals, including Huberman and Kandel (1993), Huddart (1999), DiMaggio (2015), Malliaris and Yan (2021), and Bijlsma, Boone, and Zwart (2018); we extend this line by deriving asset-pricing implications and managers' incentive to obfuscate. Our paper is closely related to Prat (2005), who shows in a general principal-agent setting that transparency can distort agents' actions away from principals' incentives, paralleling our result that transparency triggers showing-off trading. However, Prat (2005) abstracts from financial markets and treats opacity as the principal's strategic choice; our model instead derives asset-pricing implications and fragility, and analyzes obfuscation incentives from the fund manager's side.[6] Gervais and Strobl (2020) endogenizes managers' transparency choices but under exogenous costs; we instead identify endogenous opportunity costs of transparency, namely, flow-driven trading distortions, so that transparency not only sorts manager types but can generate belief-driven multiple equilibria and market fragility.

The rest of the paper proceeds as follows. Section 2 presents a baseline model with exogenous fund opacity, and Section 3 studies the equilibrium. Section 4 endogenizes opacity by allowing the manager to strategically obfuscate fund information. The Appendix contains all proofs for the theoretical results.

2 Model

This section presents a one-period trading model à la Kyle (1985) with delegated asset management.

[5]Cespa and Foucault (2014) also generate liquidity crashes from non-fundamental sources through cross-asset learning, though the mechanism differs from ours.

[6]Guerrieri and Kondor (2012), Dasgupta (2006), and Dasgupta and Prat (2008) study career concerns with endogenous asset prices, finding volatility amplification, churning, and herding respectively, but do not analyze fragility or endogenous fund opacity.

2.1 Setting

The economy consists of a *fund manager*, a competitive *market maker*, a *noise trader*, and $n \geq 2$ of *investors*. All players are risk neutral. The manager leverages her skill (defined below) to identify a single risky asset with potential mispricing, raises capital from investors, and allocates it between the risky asset and a risk-free asset with a zero interest rate.[7]

2.1.1 Skill

The manager can uncover more information about the asset's payoff than the market maker, and we interpret it as her *skill*. At the beginning of the period, the manager draws private information, s, from a normal distribution with mean zero and variance ω_s. Leveraging this information, she identifies an asset with payoff $\delta = \bar{\delta} + s$, where $\bar{\delta}$ is a publicly known mean payoff. Since the market maker does not observe s, her prior expectation of the asset's payoff is $\mathrm{E}[\delta] = \bar{\delta} + \mathrm{E}[s] = \bar{\delta}$, which is biased by s from the informed manager's perspective. As the market maker would set a price $p = \mathrm{E}[\delta] = \bar{\delta}$ in the absence of trades, s can be viewed as the asset's "potential mispricing."[8] Indeed, as shown later, the manager profits by buying the asset if $s > 0$ (underpriced) and selling it if $s < 0$ (overpriced); the larger the $|s|$, the larger the profit. In light of this, we measure the manager's *realized* asset-selection skill by $|s|$, while the variance ω_s serves as an *ex-ante* measure of the average skill.[9,10]

[7] We abstract away from information acquisition by individual investors and assume that they invest through a manager due to the lack of informational advantages. This assumption would be supported by, for example, information- or skill-acquisition costs, where only the fund manager acquires the skill at lower costs due to economies of scale (e.g., Gârleanu and Pedersen, 2018).

[8] s is not the *actual* mispricing because the market maker will take into consideration the presence of the informed manager; thus, p will deviate from $\bar{\delta}$ in equilibrium.

[9] This formalization of skill aligns with findings in the empirical studies that attribute fund managers' performance to stock-picking skills (e.g., Wermers, 2000; Kosowski, Timmermann, Wermers, and White, 2006).

[10] The definition of the average skill follows from the fact that $\mathrm{E}[|s|]$ is proportional to $\sqrt{\omega_s}$.

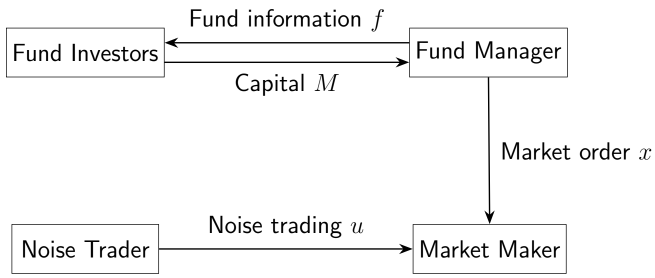


Figure 1: Capital Allocation and Investment

2.1.2 Capital Allocation and Trading

We model the manager's portfolio choice and investors' capital allocation as a rational expectations equilibrium, as illustrated in Figure 1.

Manager's portfolio choice. Upon discovering the mispricing in the asset (s), the fund manager chooses the portfolio allocation between the risky and the safe asset. This action is represented by a market order for $x \in \mathbb{R}$ units of the risky asset, submitted to the market maker in the financial market. The manager decides on x by incorporating the reactions of the financial market (i.e., the asset price) and the capital inflow from her investors, as described in the following.

Investors' capital allocation. Upon observing fund information f, each investor chooses the amount of capital to invest into the fund. Investor i's capital allocation is denoted by $m_i \geq 0$, and the total fund flow or the asset under management (AUM) of the fund is defined by $M \equiv \sum_{i=1}^{n} m_i$. The fund's fee structure is proportional to the total fund proceeds: the manager takes a $\phi \in (0, 1)$ fraction of the proceeds, while the remaining $1 - \phi$ fraction is distributed to investors in proportion to their contribution to the fund, that is, m_i/M.[11]

[11]Introducing a fixed fee does not change our qualitative results, as long as the performance-based fee also exists. More generally, a common fee structure in the hedge fund industry ("2-and-20"-style) with separate performance fee ϕ_π on π and management fee ϕ_M on M can be incorporated into our model, as it only modifies the flow-performance

Financial market. In the financial market, the market maker sets the asset price, p, based on the incoming order flow, $x+u$, where u represents a random market order from the noise trader and follows a normal distribution with mean zero and variance ω_u. Due to competition, the market maker sets a semi-strong efficient price conditional on $x+u$:[12]

$$p = \mathrm{E}[\delta | x+u]. \tag{2.1}$$

2.1.3 Obfuscation and Fund Opacity

At the fund-raising stage, the manager disseminates a private noisy signal about her trading strategy to investors, defined as $f = x + \eta$, where η represents a normally distributed noise term with mean zero and variance ω_η.[13] This signal is referred to as the *fund information*, and the noise variance of fund information, ω_η, is used as the measure of *fund opacity*. It describes intentional obfuscation of the information that investors receive about the manager's true trading strategy. A higher ω_η means that the fund information is more complex, making the strategy harder to verify or reverse engineer. This reflects the idea that managers may deliberately obfuscate details by being vague, selective, or technically complex in their descriptions, while still engaging with investors to raise capital and to comply with disclosure rules. Since the manager chooses her trading strategy x based on the private signal s, the fund information f is informative about the manager's skill and the fund's return. Thus, fund investors use f to decide on their capital allocations. In what follows, we first consider the baseline model with exogenous opacity, while Section 4 endogenizes ω_η by allowing the manager to strategically obfuscate.

sensitivity (defined below) and the fund-flow weight in the manager's objective, leaving all qualitative results unchanged.

[12] Assume that, on the equilibrium path, one competitive market maker trades the asset. As in Kyle (1985), competitively many market makers exist off the equilibrium path, ensuring a break-even price in the equilibrium.

[13] We assume that f is privately communicated between the manager and investors and is not observable to the market maker. Allowing the market maker to observe a noisy signal about x, however, does not affect the main qualitative results, as long as this signal is conditionally independent of f.

2.1.4 Maximization Problems and Equilibrium

The fund's payoff from the risky investment is δx, while it allocates $M - px$ to the risk-free asset. Hence, by denoting the return from the risky investment as $\pi = (\delta - p)x$, the total fund proceeds are represented as

$$y = \pi + M. \tag{2.2}$$

Given the realized skill s, the fund manager maximizes her expected fee income by controlling her investment strategy x:

$$\max_{x} \mathrm{E}\left[\phi y | s\right]. \tag{2.3}$$

Conversely, conditional on the fund information f, investor i maximizes her expected return from delegating capital management:[14]

$$\max_{m_i \geq 0} \mathrm{E}\left[\frac{m_i}{M}(1-\phi)y - m_i | f\right], \tag{2.4}$$

where the first term represents the expected payment from the fund, and the second term is the cost of capital.

Definition 1. *The equilibrium is defined by the market maker's pricing strategy p, the fund manager's trading strategy x, and investors' capital allocations $\{m_i\}_{i=1}^n$, such that, (i) p satisfies (2.1) given $x + u$, (ii) x solves (2.3) given s, p, and $\{m_i\}_{i=1}^n$, and (iii) for all i, m_i solves (2.4) given p, f, and $\{m_j\}_{j \neq i}$.*

[14]The fund manager may have an incentive to make a false report about y to her investors. Namely, by reporting $\tilde{y}$, she pays out $(1-\phi)\tilde{y}$ to investors and captures $y-(1-\phi)\tilde{y}$ on her own. To prevent this intentional misreporting, we assume that each investor can verify x after the trading stage. Alternatively, we may assume costly verification: investors collectively pay the verification cost C to back out x from the fund information in the end of the trading period. As long as C positively depends on the degree of fund opacity ω_η, our qualitative results remain the same.

3 Equilibrium

In this section, we search for a linear equilibrium in which the manager's market order x is linear in her skill s, and the market maker's pricing strategy p is also linear in the order flow $x + u$. Proofs for the analytical results are provided in the Appendix.

3.1 Trading Strategy

We conjecture and later verify that the manager's equilibrium trading strategy is

$$x(s) = \beta s, \tag{3.1}$$

where $\beta > 0$ is an endogenous constant determined in the equilibrium. Conjecture (3.1) states that the manager buys (sells) the asset if her s is positive (negative), where the order size is proportional to her skill $|s|$; the more skilled the manager, the more aggressively she trades.[15] The scale factor β is referred to as the *trading intensity*, representing how intensively the manager exploits her informational advantage over the market maker.

3.2 Execution Price

The market maker decides on the price of the asset by extracting information about the asset's payoff from the aggregate order flow $x+u$. Anticipating the trading strategy (3.1), the price in equation (2.1) can be rewritten as

$$p = \bar{\delta} + \lambda(x + u), \tag{3.2}$$

where

$$\lambda \equiv \frac{\beta \omega_s}{\beta^2 \omega_s + \omega_u} \tag{3.3}$$

[15]This conjecture follows from the original Kyle model where an informed trader trades based on the difference between her belief and that of the market maker, i.e., $\mathrm{E}[\delta|s] - \mathrm{E}[\delta] = s$.

is referred to as "Kyle's lambda," a measure of the price impact of order flow.[16] Following the literature, we define a market to be *liquid* if λ is small.

3.3 Investors' Capital Allocation

By incorporating the fund proceeds $y = \pi + M$, investor i's optimization problem in (2.4) is

$$\max_{m_i \geq 0} \frac{m_i}{M}(1-\phi)\mathrm{E}\left[\pi|f\right] - \phi m_i. \tag{3.4}$$

Investor i solves this problem by incorporating the impact of her choice (m_i) on the total fund flow (M), while taking other investors' behavior ($\sum_{j \neq i} m_j$) as given. As she allocates a larger amount of capital, its marginal return diminishes due to the pro-rata allocation of the fund's proceeds, while the cost of capital linearly increases. This structure ensures the second-order condition of problem (3.4), and the first-order condition pins down the optimal capital allocation as follows:

$$m_i = \left(\frac{1-\phi}{\phi}\mathrm{E}[\pi|f]\sum_{j \neq i} m_j\right)^{\frac{1}{2}} - \sum_{j \neq i} m_j. \tag{3.5}$$

We focus on the symmetric equilibrium where all investors take the same action, $m_i = m$ for all $i = 1, 2, \cdots, n$. In this equilibrium, equation (3.5) reduces to

$$m = \frac{n-1}{n^2}\frac{1-\phi}{\phi}\mathrm{E}[\pi|f], \tag{3.6}$$

and the total fund flow becomes

$$M \equiv nm = \theta \mathrm{E}[\pi|f], \tag{3.7}$$

where $\theta \equiv \frac{n-1}{n}\frac{1-\phi}{\phi}$ represents the *flow-performance sensitivity*. Both the individual capital allocation and the total fund flow are linearly increasing in the

[16] Since the market maker believes that the manager follows (3.1), the linear filtering rule with normally distributed random variables yields $p = \mathrm{E}[\bar{\delta} + s|\beta s + u] = \bar{\delta} + \mathrm{E}[s] + \frac{\mathrm{Cov}[s, \beta s + u]}{\mathrm{Var}[\beta s + u]}(x + u - \mathrm{E}[\beta s + u]) = \bar{\delta} + \frac{\beta \omega_s}{\beta^2 \omega_s + \omega_u}(x + u)$.

expected trading profit conditional on the fund information, $\mathrm{E}[\pi|f]$. This is due to the performance-based fee structure, which makes the per-unit return to investors increasing in trading profits. The fund flow becomes more sensitive to the expected performance (θ increases) when the fee rate (ϕ) is low, as it reduces the cost of delegation and encourages investment; and when the fund has a large clientele (n), as it attracts capital from a larger number of investors.

3.4 Reputation

In determining the capital allocation in (3.6), each investor seeks to compute the expected fund profits $\mathrm{E}[\pi|f]$. This process, in turn, requires the investor to back out the realized skill level of the manager $|s|$ from fund information $f = x + \eta$.

Lemma 3.1. *Upon observing the fund information $f = x+\eta$, investors update their expectation about the trading profit as*

$$\mathrm{E}\left[\pi|f\right] = (1-\lambda\beta)\beta\mathrm{E}[s^2|f] = (1-\lambda\beta)\beta\left(\hat{s}^2 + \hat{\omega}_s\right), \tag{3.8}$$

where the posterior expectation of the (signed) realized skill is

$$\hat{s} = \mathrm{E}[s|f] = \xi(x+\eta), \tag{3.9}$$

with

$$\xi = \frac{\beta\omega_s}{\beta^2\omega_s + \omega_\eta}, \tag{3.10}$$

and the posterior of the skill variance is

$$\hat{\omega}_s = \mathrm{Var}[s|f] = \frac{\omega_s\omega_\eta}{\beta^2\omega_s + \omega_\eta}. \tag{3.11}$$

Equation (3.8) suggests that the fund return, $\pi = (\delta - p)x$, becomes a quadratic function of s, as both the manager's trading quantity x and the profit margin $\delta - p$ are proportional to s in expectation. Therefore, investors seek to update their belief about the manager's skill based on fund information. Equation (3.9) describes investors' belief-updating process: $\hat{s}$ represents the updated expectation about skill and is referred to as the *reputation* in the following discussion. According to equation (3.9), ξ measures how much investors rely on f in updating their belief and governs the sensitivity of the reputation to the fund's trading strategy x. Holding β constant, ξ is large when investors' prior is unreliable (ω_s is large) and the fund information is transparent (ω_η is small). More importantly, ξ itself also depends on the trading strategy β, as a more intensive trading strategy makes f more informative about s.

Lemma 3.1 implies that, on top of maximizing profits, delegated asset management generates an additional incentive for the manager to control her trading strategy x, as it affects the total fund flow M by influencing investors' expectations about fund returns.

3.5 Manager's Optimization

By incorporating the total fund flow ([3.7] and [3.8]), the manager's problem in (2.3) is rewritten as follows:

$$\max_x \; sx - \lambda x^2 + \theta(1-\lambda\beta)\beta\left[\xi^2(x^2+\omega_\eta)+\hat{\omega}_s\right]. \tag{3.12}$$

The first two terms represent the fee income from the investment return and are essentially identical to the trading profit in the original Kyle (1985) model. The last term with coefficient θ arises from the fee income associated with the fund flow M. It increases with the manager's risk taking, measured by x^2, because she can potentially inflate her reputation $|\hat{s}|$ by placing a large order (either buy or sell). The reaction of the expected fund flow to the trading strategy is influenced not only by the flow-performance sensitivity θ but also by the investors' updating coefficient ξ. This is because fund opacity ($\omega_\eta > 0$) prevents investors from perfectly inferring the manager's trading strategy from fund information. Importantly, investors' estimate of the manager's skill and, therefore, the reaction of the fund flow to the risky investment depend on the risky trading strategy β itself: the more intensively the manager trades, the more reliable the fund information becomes for investors. This structure is captured by the dependence of ξ on β in equation (3.10) and leads to the strategic complementarity between the manager's risk taking and investors' capital allocation based on reputation.

The unique solution to the manager's optimization problem in (3.12) (given β) is represented by

$$x = F(\beta)s, \tag{3.13}$$

where

$$F(\beta) \equiv \frac{1}{2\left(\lambda - \theta(1-\lambda\beta)\beta\xi^2\right)}. \tag{3.14}$$

$F(\cdot)$ is a nonlinear function of β due to the dependence of λ and ξ on β.

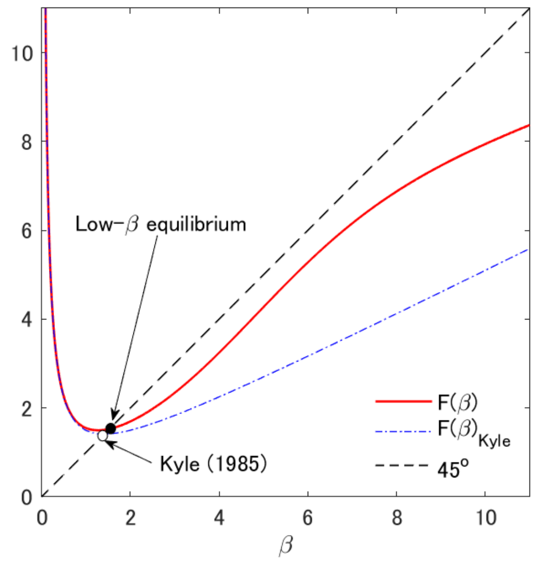
3.6 Equilibrium under Opacity

The manager optimally chooses (3.13) given that the market maker and investors believe that she follows strategy (3.1). In equilibrium, their beliefs must be correct, meaning that (3.1) and (3.13) must be consistent with each other. This belief consistency condition requires that

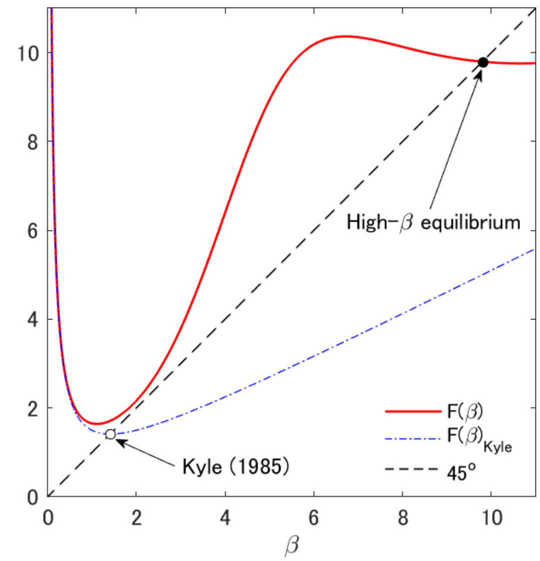
$$\beta = F(\beta). \tag{3.15}$$

That is, the equilibrium levels of β are determined at the fixed points of $F(\cdot)$. Having identified β, the equilibrium levels of x, p, and M are determined by equations (3.1), (3.2) and (3.7), respectively. In what follows, we first illustrate the determination of equilibrium β through a numerical example, followed by an analytical proposition that formally establishes the equilibria.

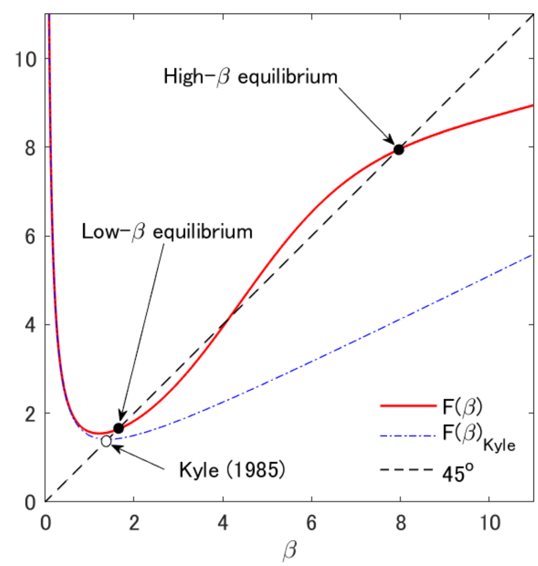
Figure 2 illustrates the determination of β for three different levels of fund opacity ω_η. The intersections of $F(\beta)$ (the solid line) and the 45-degree line yield the equilibrium levels of β. We also plot $F(\beta)_{\text{Kyle}}$ in the dash-dotted line, representing the original Kyle (1985) model. This benchmark case arises when the fund is infinitely opaque (i.e., $\omega_\eta \to \infty$), as it makes the fund flow inelastic to the manager's trading strategy, and the delegated asset management



(a): Large ω_η ($\omega_\eta = 48$) $\Rightarrow$ unique low-β equilibrium.



(b): Small ω_η ($\omega_\eta = 28$) $\Rightarrow$ unique high-β equilibrium.



(c): Intermediate ω_η ($\omega_\eta = 38$) $\Rightarrow$ multiple high-β & low-β equilibria.

Figure 2: Determination of β

Note: The figure plots $F(\beta)$ for different values of θ. The equilibrium values of β are determined at the fixed points of $F(\cdot)$. The parameter values used in the figures are $\omega_s = 1$, $\omega_u = 2$, $n = 5$, and $\phi = 0.02$.

becomes irrelevant.

Panel (a) of Figure 2 shows that if ω_η is large, there is a unique "low-β equilibrium," in which β is close to the trading intensity without delegated asset management. Panel (b) shows that a small ω_η leads to a unique "high-β equilibrium," in which β is much larger than that in the original Kyle (1985) model. Panel (c) shows that multiple equilibria arise for an intermediate level of ω_η. Specifically, the high-β and the low-β equilibria are stable, as $F(\beta)$ crosses the 45-degree line from above, whereas the one with intermediate β is unstable, as $F(\beta)$ crosses the line from below.[17] In what follows, we focus only on the stable equilibria.

Fund opacity and multiple equilibria. To see the intuition behind the different equilibrium patterns presented in Figure 2, note that the fund manager faces two conflicting motives when trading. On the one hand, she seeks to avoid trading too aggressively on her private information, as doing so would move the price excessively and reduce her trading profits. As in Kyle (1985), this motive leads the manager to decrease β. On the other hand, she wishes to trade intensively to signal her skill to her investors, aiming to establish a strong reputation and attract large fund flows. This motive encourages her to increase β. In summary, she faces a tradeoff between "hiding" her skill from the market maker and "showing it off" to her investors. The fund opacity ω_η governs this tradeoff by influencing the sensitivity of the manager's reputation held by investors to her investment behavior. In particular, Lemma 3.1 demonstrates that the fund flow M becomes less sensitive to x under opacity (i.e., ξ is small) compared to the fully transparent case, as it becomes difficult for investors to estimate the skill based on the fund information f. Therefore, fund opacity diminishes the strategic complementarity between the manager's risk taking and the reputation-driven capital allocation by investors, thereby weakening the showing-off motive.

[17]We define an equilibrium to be stable (unstable) if the corresponding β is a stable (unstable) fixed point of $F(\cdot)$, that is, $|F'(\beta)| < 1$ (> 1). A stable equilibrium is robust to a small perturbation to the agents' belief about β, meaning that the economy converges back to the original equilibrium through the best-response dynamics of the agents' beliefs.

For an opaque fund with a large ω_η (panel [a]), the hiding motive dominates the showing-off motive, leading to a unique low-β equilibrium. In this equilibrium, investors believe that the manager's trading intensity is relatively low and the fund information is not very informative, making the reputation and the fund flow insensitive to x. Although the manager could inflate her reputation by placing an order larger than what her investors anticipate, she refrains from doing so, because it would cause a large price impact and reduce the fund's investment profits, while fund flows and her fee income would not increase much due to small ξ. Consequently, the equilibrium β is close to the trading intensity without delegated asset management, meaning that it is a "Kyle-like equilibrium."

For a transparent fund with a small ω_η (panel [b]), the showing-off motive prevails, resulting in a unique high-β equilibrium. In this equilibrium, investors believe that the manager is trading aggressively. The reputation and fund flows grow sensitive to the manager's trading strategy through the fund information. Under such beliefs, the manager is actually willing to trade aggressively: although it moves the price substantially and undermines investment profits, it helps her build a strong reputation and generates a large fee income from significant capital inflows. Importantly, investors' learning is not manipulated, as their belief about the manager's trading strategy is correct. Nonetheless, the manager places a large order because doing otherwise would induce her investors to incorrectly estimate that she is less skilled, thereby deteriorating her reputation. She trades aggressively solely to conform to her investors' belief and to attract large fund flows. To emphasize this mechanism, we call this equilibrium the "flow-driven equilibrium."

For intermediate levels of ω_η (panel [c]), the strategic complementarity between investors' reputation-based capital allocation and the manager's risk taking is neither weak nor strong, and there is no clear dominance between the two motives. Consequently, both the low-β and the high-β equilibria are supported under intermediate fund opacity. Each equilibrium is associated with a self-fulfilling belief of the market maker and investors.[18]

[18]For simplicity, we assume that agents do not agree to disagree, so that they coordinate

The following proposition summarizes the above discussion and provides the analytical characterization of the equilibria.

Proposition 3.1 (Equilibrium characterization)**.** *There exists a critical value of the flow-performance sensitivity, denoted as* θ_0*.*

(i) If $\theta \leq \theta_0$*, the equilibrium is unique and characterized by a threshold of fund opacity* ω_0*, such that,* $\omega_\eta \leq \omega_0$ *leads to the high-*β *equilibrium and* $\omega_\eta \geq \omega_0$ *leads to the low-*β *equilibrium.*

(ii) If $\theta > \theta_0$*, there are two thresholds of fund opacity,* ω_L *and* $\omega_H (> \omega_L)$*. When* $\omega_\eta \leq \omega_L$*, the high-*$\beta$ *equilibrium is unique; when* $\omega_\eta \geq \omega_H$*, the low-*$\beta$ *equilibrium is unique; and when* $\omega_\eta \in (\omega_L, \omega_H)$*, the high- and low-*$\beta$ *equilibria coexist.*

Figure 3 illustrates the result in Proposition 3.1. It indicates that both fund opacity ω_η and flow-performance sensitivity θ are critical in determining the equilibrium type, as they jointly influence the reaction of the fund flow to changes in the manager's trading strategy, thereby affecting her showing-off motive. As high levels of θ strengthen this motive, we obtain the following corollary:

Corollary 3.1. *(i) When* $\theta \leq \theta_0$*, the threshold of fund opacity* ω_0 *is monotonically increasing in* θ *with* $\lim_{\theta \to 0} \omega_0 = 0$*.*

(ii) When $\theta > \theta_0$*, both* ω_L *and* ω_H *are increasing in* θ*. Moreover, the region for multiple equilibria,* (ω_L, ω_H)*, expands as* θ *increases.*

For a given value of fund opacity ω_η, a weak flow-performance sensitivity supports only the low-β equilibrium, as the hiding motive strictly dominates the weak showing-off motive. However, as θ increases, a high flow-performance sensitivity encourages the manager to show off her skill by trading intensively. Hence, the high-β equilibrium emerges. The two motives are balanced and generate multiple equilibria when the fund is moderately opaque and fund flows are sensitive to the expected investment performance. Otherwise, one

their belief when parameter values are such that multiple equilibria exist.

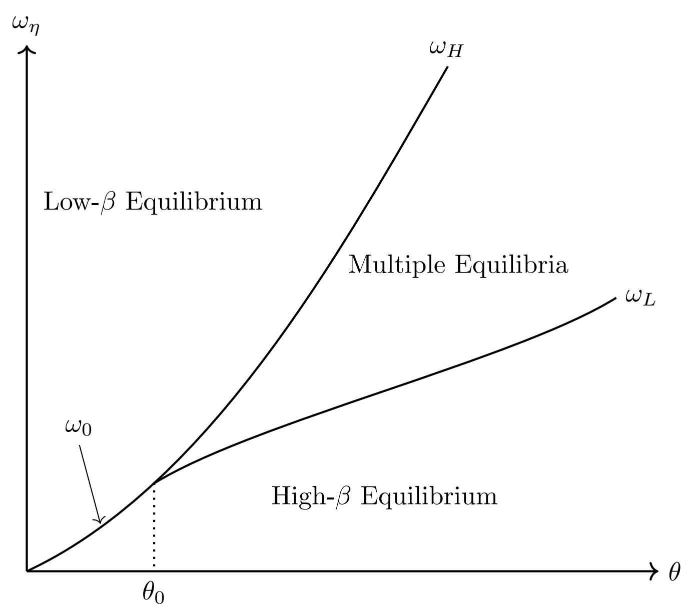


Figure 3: Equilibrium Characterization

Note: This figure illustrates the equilibrium states using the flow-performance sensitivity (θ) and fund opacity (ω_η). When $\theta \leq \theta_0$, the cutoffs of opacity converge to $\omega_H = \omega_L = \omega_0$.

of the conflicting motives dominates the other, making either the Kyle-like equilibrium or the flow-driven equilibrium the unique equilibrium.

Equilibrium comparison. Comparing the two stable equilibria, does the manager perform better in one of them? What are the implications for market liquidity, asset prices, and learning about asset-selection skills?

Corollary 3.2 (High-β vs. low-β equilibrium)**.** *In the high-β equilibrium, relative to the low-β equilibrium, the following results hold.*

(i) The market is more liquid, that is, λ is lower;

(ii) The manager's expected investment performance given the realized skill, $\mathrm{E}[(\delta - p)x|s] = \frac{\beta\omega_u s^2}{\beta^2\omega_s+\omega_u}$*, is lower;*

(iii) The price informativeness, defined by $\tau \equiv \frac{1}{\mathrm{Var}[\delta|p]} = \frac{\beta^2\omega_s+\omega_u}{\omega_u\omega_s}$*, is higher;*

(iv) The price volatility, $\mathrm{Var}[p] = \frac{\beta^2\omega_s^2}{\beta^2\omega_s+\omega_u}$*, is higher; and*

(v) Investors' estimate of the manager's skill is more precise, that is, $\hat{\omega}_s = \mathrm{Var}[s|f] = \frac{\omega_s\omega_\eta}{\beta^2\omega_s+\omega_\eta}$ *is lower.*

Statement (i) of Corollary 3.2 holds because price impact λ is decreasing in β in the equilibrium. Namely, the information is exaggerated in that the order size is too large relative to the true $|s|$ due to the showing-off motive. Thus, the market maker attempts to partially "undo" the manager's information revelation by incorporating less of it into the price, resulting in a weaker price impact.[19] Since the high-β equilibrium emerges only if the fund is relatively transparent, the statement also implies that a higher fund transparency leads to higher market liquidity.

The intuition for statement (ii) is closely related to that for statement (i). In the high-β equilibrium, the manager earns a lower trading profit due to a smaller informational advantage over the market maker. This result may appear inconsistent with statement (i) because, ceteris paribus, a lower price

[19] λ represents the price's reaction to the order flow $x + u$, rather than its reaction to the manager's private information s. The price impounds s with coefficient $\beta\lambda$, which is monotonically increasing in β.

impact is typically associated with a higher trading profit. However, the manager's order size $|x|$ in the high-β equilibrium is so large that the *overall* price reaction (i.e., λ multiplied by $|x|$) is larger than that in the low-β equilibrium. Consequently, the manager moves the price excessively and earns a lower trading profit in this equilibrium. Due to her aggressive trading, more information about s, which is informative about δ, is incorporated into the price in the high-β equilibrium, supporting statement (iii).

Statement (iv) follows, again, from the manager's aggressive trading in the high-β equilibrium. Specifically, she places a large buy (sell) order when $s > 0$ ($s < 0$), even if $|s|$ is small. This causes a large upward (downward) pressure on p, leading to greater price variability. This result aligns with the insight of Guerrieri and Kondor (2012) that reputation concerns amplify price volatility, although their mechanism differs from ours.[20]

In the high-β equilibrium, the manager reveals more information about s, thereby improving the precision of investors' estimation, as statement (v) demonstrates. This makes fund flows more responsive to skill, as investors can assess it more accurately.

3.7 Implications

This section discusses economic implications drawn from the equilibrium analyses under opacity.

3.7.1 Financial Fragility

The self-fulfilling nature of multiple equilibria suggests that financial markets may become vulnerable to non-fundamental factors. A shift in investor beliefs alone, without any structural changes, could cause the market to transition from the standard Kyle-like equilibrium to the flow-driven equilibrium, characterized by high price volatility and poor trading performance. We interpret this phenomenon as a form of *fragility* in financial markets.

[20] Guerrieri and Kondor (2012) argue that bond price volatility increases because fund managers demand a premium to offset the risk of reputation damage in the event of default.

Our model suggests that fragility may occur when flow-performance sensitivity (θ) is sufficiently high. This result is consistent with empirical studies documenting that strong flow-performance sensitivity can amplify trading pressure and contribute to market fragility by inducing managers to adjust trading behavior in response to investor flows rather than fundamentals (e.g., Coval and Stafford, 2007; Goldstein, Jiang, and Ng, 2017). While this literature typically attributes fragility to flow dynamics without explicitly focusing on the size of the investor base, our results suggest that a larger clientele (n) mechanically increases θ and thereby sows the seeds of market fragility.

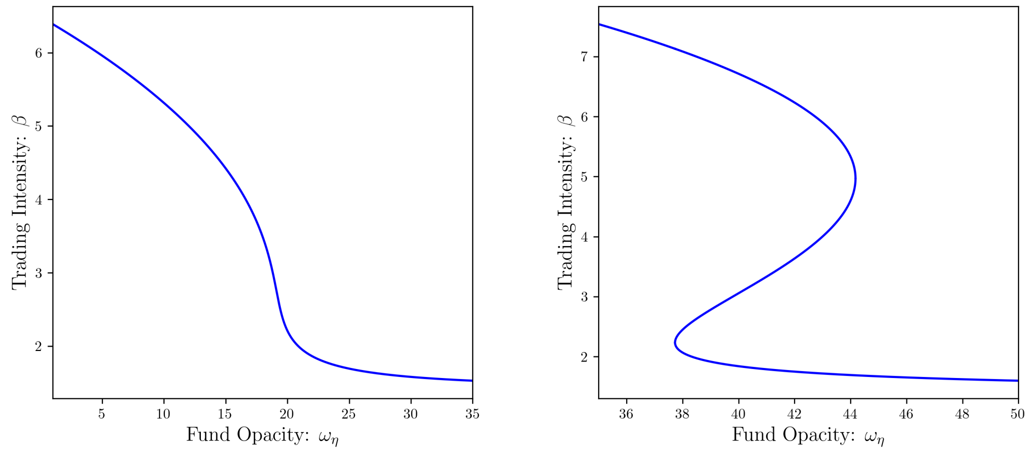
A high flow-performance sensitivity can also arise when the fund management fee (ϕ) is relatively low. This surprising result goes counter to the recent debate on performance-based compensations for fund managers and advisory contracts, where payment structures that strongly reward performance are often criticized for encouraging excessive risk-taking. In our model, such performance-based compensation can actually stabilize the market by eliminating the flow-driven equilibrium and guiding the economy toward a unique low-β equilibrium. This is consistent with the empirical finding by Dass, Massa, and Patgiri (2008) that high-powered advisory contracts do not induce investment in bubbly stocks but instead reduce such investments, highlighting the stabilizing effect of incentive compensations on financial markets.

3.7.2 Opacity and Trading Style

Furthermore, Proposition 3.1 yields novel implications connecting the fund opacity and trading styles.

Corollary 3.3. *Both in the high- and low-β equilibria, the trading intensity β is monotonically decreasing in the fund opacity ω_η.*

Figure 4 illustrates the result in Corollary 3.3: the left panel represents the case with a unique equilibrium by setting a weak flow-performance sensitivity, while the right panel involves multiple equilibria due to a strong flow-performance sensitivity. First, as both panels illustrate, opaque funds tend to adopt low-intensity strategies (i.e., the low-β equilibrium) that minimize



(A) Low Flow-performance Sensitivity

(B) High Flow-performance Sensitivity

Figure 4: Fund Opacity and Trading Intensity

Note: These figures are illustrated by seeting $\omega_s = 1.0, \omega_u = 2.0, \theta = 10(\phi = 0.08)$ for the left panel and $\theta = 27(\phi = 0.02)$ for the right panel.

market impact, prioritizing profits over fund flows. In contrast, transparent funds engage in high-intensity strategies with flashy risk taking (i.e., the high-β equilibrium), seeking reputation among investors and large fund flows. This relationship arises because the manager's reputation becomes insensitive to her trading behavior when the fund information is opaque and difficult to interpret for fund investors. The model predicts that these tendencies are more pronounced when the flow-performance sensitivity is weak, as the equilibrium is unique regardless of fund opacity (panel [a] of Figure 4).

These results are consistent with common observations in the real market. For example, quantitative hedge funds are often extremely opaque and highly profit-driven, focusing on consistent performance, with benefits from attracting fund flows being regarded as secondary to profitability (Schwager, 2012). On the other hand, some transparent funds, such as certain thematic ETFs or crypto-focused funds, engage in bold, attention-grabbing strategies to attract investor flows. These funds often emphasize visibility and narrative-driven positioning, which can lead to volatile returns and greater risk taking.

The second implication is drawn from the model's multiple equilibria, as panel (b) in Figure 4 illustrates. Namely, the relation between funds' activities (β) and their transparency (ω_η) becomes indeterminate when the opacity is at an intermediate level. Notably, this indeterminacy may lead to financial fragility. Depending on market beliefs, funds with intermediate opacity may shift between profit-seeking and flow-driven strategies, creating non-fundamental fluctuations. Our result suggests that even relatively opaque funds may follow the crowd due to market beliefs, switching between safe and risky strategies.

A notable observation in line with this result is momentum crashes (Daniel and Moskowitz, 2016), where funds engaging in risky and visible momentum trades often switch to safer strategies, generating sharp reversals and amplifying price declines. More generally, the rapid shifts between risk-taking and solid performance-based behavior have contributed to market fragility and volatility, as documented in various studies on economic crises, such as Brunnermeier (2009) on the Global Financial Crisis in 2007. Importantly, large fundamental shifts that support such transitions are rarely identified (Gennotte and Leland, 1999). Our model does not require such fundamental shocks, as a change in beliefs alone can trigger a shift.

3.7.3 Compensation in Fund Management Industry

Our model provides insights into how skills are reflected in compensation in the fund management industry. As shown in statement (v) of Corollary 3.2, fund investors learn s more precisely in the high-β equilibrium than in the low-β equilibrium, as the manager demonstrates her skill by trading more intensively. Consequently, in the high-β equilibrium, the manager's expected compensation, $\mathrm{E}[\phi y|s]$, becomes more sensitive to her realized skill. As an illustration, Figure 5 plots the manager's expected compensation in relation to her skill in the low- and high-β equilibria. In the high-β equilibrium, investors' accurate evaluations result in greater pay inequality: low-skilled managers would receive very low compensation, while high-skilled ones are very highly paid. This result corroborates empirical observations, such as those provided by Philippon and Reshef (2012) and Böhm, Metzger, and Strömberg (2018),

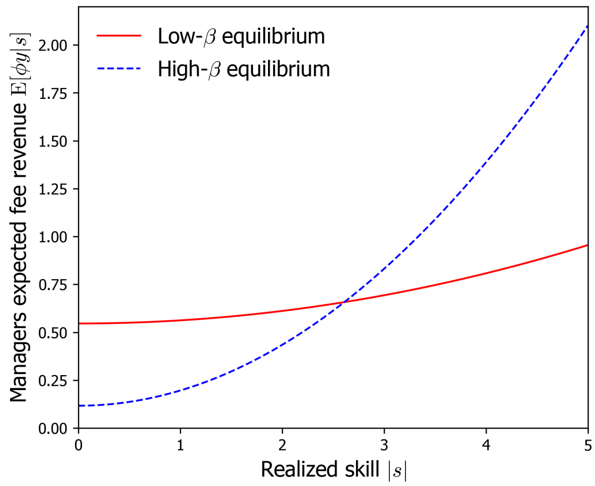


Figure 5: Skill versus Expected Wage

Note: The figure plots the manager's expected fee income at the low-β equilibrium (solid line) and high-β equilibrium (dashed line) for different levels of skill, $|s|$. The parameter values used in the figures are $\omega_s = 1$, $\omega_u = 2$, $\omega_\eta = 38$, and $\theta = 27$.

highlighting the comparatively large wage inequality in the fund management industry. Célérier and Vallée (2019) attribute the source of wage inequality to the scalability of skill, while inequality in our model arises from managers' incentives to attract fund flows through establishing high reputation.

3.7.4 Skill and Performance Persistence

Do high-skilled managers consistently perform well? Not always: skilled managers may not necessarily outperform less skilled ones. Consider a highly skilled manager and a relatively less skilled one. Since trading profits are increasing in managers' skill $|s|$, the former is expected to achieve higher trading profits than the latter *conditional on* both managers trading at the same intensity β. However, if their investors believe, for whatever reason, that the skilled manager invests more aggressively than the less skilled one, and managers conform to such beliefs, a high skill level does not translate into performance. This is because the skilled manager, trading at the flow-driven equilibrium, reveals too much private information to the market maker. Hence, compared to the

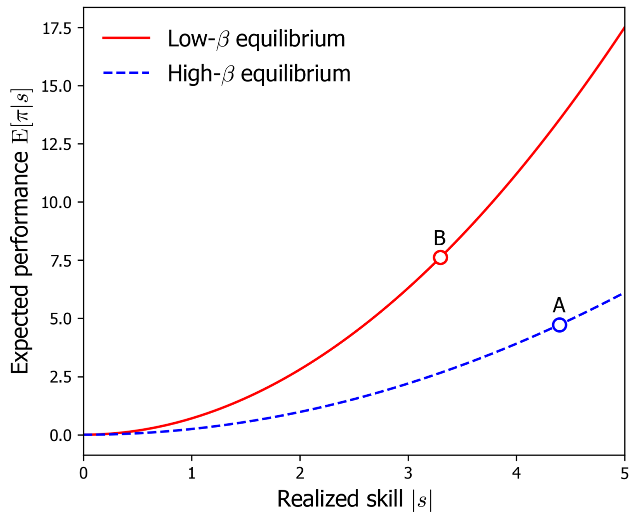


Figure 6: Skill versus Expected Trading Profit

Note: The figure plots the expected trading profit $E[\pi]$ at the low-β equilibrium (solid line) and the high-β equilibrium (dashed line) for different levels of skill $|s|$. The parameter values used in the figure are $\omega_s = 1$, $\omega_u = 2$, $\omega_\eta = 38$, and $\phi = 0.02$.

less skilled manager, who plays the Kyle-like equilibrium, the skilled manager may earn lower expected profits. Figure 6 illustrates such a situation.[21] It plots the expected trading profits (performance) at high- and low-β equilibria with different levels of skill, suggesting that a low-skilled manager in the low-β equilibrium (point B) outperforms a high-skill manager in the high-β equilibrium (point A).

The empirical literature (e.g., Jensen, 1967; Busse, Goyal, and Wahal, 2010) finds that asset managers do not outperform passive benchmarks on average and, in cross-section, those who outperform cannot do so consistently. While the literature attributes the lack of persistent performance to the lack of skill, our result proposes an alternative explanation. Several studies reconcile the existence of skill with the lack of persistent performance. Berk and Green (2004), for example, argue that skill-based alphas eventually disappear due to competition between investors and decreasing returns to scale at the fund

[21]The same argument holds for the comparison of the unconditional expected profit, $E[\pi]$, using skill variance ω_s as an *ex-ante* measure of the skill level.

level.[22] Our model contributes to the understanding of this matter by showing that the absence of a systematic skill-performance relationship could be the result of self-fulfilling multiple equilibria.

4 Strategic Obfuscation

This section analyzes endogenous fund opacity ω_η. Our results shed light on why managers obfuscate and which types of managers contribute to financial fragility.

4.1 Setup

Obfuscating fund information. Prior to investors' capital allocation and the manager's investment decision, the fund manager chooses the fund opacity ω_η, anticipating the equilibrium reactions of investors and the market maker specified in Section 3. For example, in many mutual funds, managers provide information about their strategies to investors in accordance with regulatory requirements. However, they strategically design the way this information is presented and convey a deliberately uninformative description even when the underlying holdings are simple. We assume that such practices entail costs: the manager pays the obfuscation cost, $C(\omega_\eta)$ with $C'(\cdot) > 0$ and $C(0) = C'(0) = 0$, to increase the degree of opacity of fund information. This cost can be interpreted as pecuniary costs arising from investments in communication and marketing resources to design abstract narratives and vague labels that replace direct descriptions of risky portfolios. It may also reflect non-pecuniary effort that managers expend in carefully structuring disclosure documents to maintain opacity while following regulatory compliance.

Sunspot shock. To formalize the manager's optimization problem in the rational expectations framework, we introduce an equilibrium-selection device

[22] Pastor, Stambaugh, and Taylor (2015) document increases in skill in finance and attribute the lack of corresponding increases in performance to decreasing returns to scale at the industry level.

in cases of multiple equilibria. In particular, following the most common approach, we assume that market participants condition their actions on the realization of an exogenous *sunspot* shock, characterized by the binary random variable $z \in \{0, 1\}$.[23] Namely, whenever parameters support multiple equilibria, market participants play the high-β equilibrium if a spot appears on the sun ($z = 1$), whereas the low-β equilibrium is realized if no spots are observed ($z = 0$). Accordingly, the high- and low-β equilibria are realized with probability $\rho_H \equiv \Pr(z = 1)$ and $\rho_L = 1 - \rho_H$, respectively. The shock is realized after the fund manager sets fund opacity but prior to the fund-raising stage.

Manager's expected utility. The manager controls ω_η to maximize her *ex-ante* expected utility derived from the fee income $\mathrm{E}[\phi y]$, net of the obfuscation cost C. As the fund flow is given by equation (3.7), her objective function is specified as follows.[24]

$$U = \phi \mathrm{E}[\pi + M] - C(\omega_\eta) = \phi(1 + \theta)\mathrm{E}[\pi] - C(\omega_\eta), \tag{4.1}$$

where the expectation operator incorporates the sunspot shock together with all other random variables introduced so far.

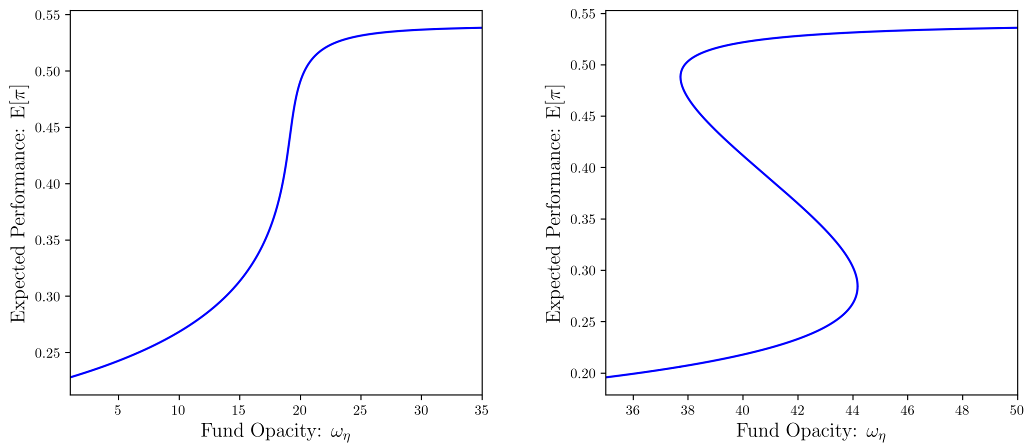
Equilibrium. In this extended model, the definition of equilibrium augments Definition 1 of the baseline model by additionally requiring that ω_η maximizes the manager's objective function U in equation (4.1).

4.2 Opacity as Commitment Device

Before solving problem (4.1), consider the manager's obfuscation incentive. Given the trading intensity β, the unconditional expectation of fund perfor-

[23]The sunspot equilibrium is proposed by Diamond and Dybvig (1983) as an equilibrium selection device in the context of bank runs and formalized in the equilibrium analyses by the subsequent studies, such as Cooper and Ross (1998).

[24]The law of iterated expectation implies $\mathrm{E}[M] = \theta \mathrm{E}\left[\mathrm{E}[\pi|f]\right] = \theta \mathrm{E}[\pi]$.



(A) Low Flow-performance Sensitivity

(B) High Flow-performance Sensitivity

Figure 7: Fund Opacity and Expected Performance

Note: The figure plots the unconditional expected performance of the fund $\mathrm{E}[\pi]$ against fund opacity ω_η with different values of the flow-performance sensitivity: $\theta = 10$ (the left panel) and $\theta = 27$ (the right panel). The value of other parameters are $\omega_s = 1.5$ and $\omega_u = 3$.

mance in (4.1) is computed as

$$\mathrm{E}[\pi] = \frac{\beta \omega_u}{\beta^2 \omega_s + \omega_u} \omega_s. \tag{4.2}$$

As in the standard Kyle (1985) framework, the first fraction represents the profit margin that each unit of informational advantage would earn, and the second term ω_s represents the average magnitude squared of the informational advantage.

Fund opacity ω_η does not directly show up in equation (4.2), but it influences the expected performance through the equilibrium trading intensity β, as Corollary 3.3 demonstrates.

Corollary 4.1. *Both in the low- and high-β equilibria, the expected fund performance* $\mathrm{E}[\pi]$ *is monotonically increasing in fund opacity* ω_η.

Figure 7 plots $\mathrm{E}[\pi]$ in relation to ω_η, which traces the reactions of β in Figure 4. In our model, holding other parameters fixed, $\mathrm{E}[\pi]$ declines with the trading intensity β. This arises from the presence of the showing-off motive. Although unconditional expected performance is maximized at the Kyle benchmark, which is driven solely by the hiding motive, the manager in our model trades more aggressively in order to attract fund flows. It reveals too much private information to the market maker and lowers the fund performance, while the manager is willing to deviate from the profit-maximizing strategy because the fund flow compensates for this loss. Fund opacity, conversely, diminishes the showing-off motive and induces the manager to pursue more profit-driven strategies, bringing β closer to the profit-maximizing level. As a consequence, opacity helps improve the expected fund performance and, according to the manager's utility (4.1), this effect encourages the manager to obfuscate fund information.

Interestingly, investors' *ex-ante* expected utility is also increasing in fund opacity. According to equation (3.6), each investor expects to obtain

$$U_i \equiv \frac{1-\phi(1+\theta)}{n}\mathrm{E}[\pi], \tag{4.3}$$

so that her expected utility is proportional to the fund's unconditional expected performance. Thus, Corollary 4.1 implies that even investors prefer opaque fund from the *ex-ante* perspective. This result may appear paradoxical in light of the showing-off motive: why does the manager trade aggressively and deviate from the performance-maximizing strategy, despite knowing that such behavior lowers performance and reduces both her own and investors' expected utility? This tension arises from a crucial difference in timing. When she decides on the trading strategy x, the manager cannot alter the market's beliefs about her strategy (β and λ in equations [3.8]–[3.11]), meaning that she optimizes over x by taking the investors' belief-updating rule as given. Since she can influence investor perceptions only through the content of fund information f, she is inclined to inflate her trading volume $|x|$ to appear more skilled, as captured by the showing-off motive. Conversely, at the *ex-ante* stage (i.e., before she sends the fund information to investors), the manager recognizes that this future showing-off motive will lead to flashy but inefficient risk-taking that erodes unconditional performance and the expected fund flow.

Consequently, she strategically chooses fund opacity as a commitment device to "tie her own hands," thereby dampening the investors' future responsiveness to f and inducing her future self to prioritize profit-driven trading over flow-driven trading.

4.3 Equilibrium Opacity

We now solve for the manager's optimal opacity, ω_η^*, that maximizes U in (4.1) and characterize how it varies with skill.

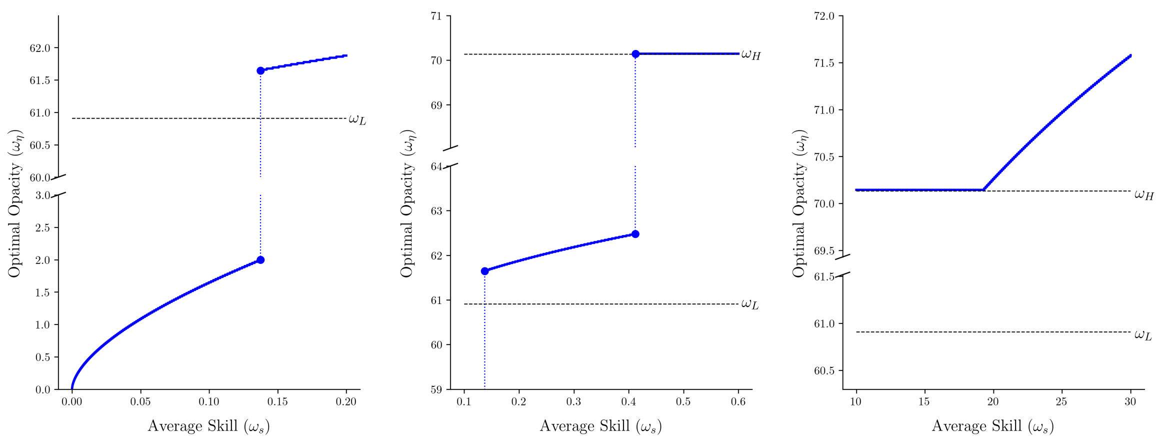
Proposition 4.1. *The optimal opacity ω_η^* is positive for all $\omega_s > 0$ and non-decreasing in ω_s. Moreover, it is strictly increasing in ω_s whenever the solution is interior to each equilibrium regime.*

Figure 8 presents a numerical illustration of Proposition 4.1, additionally revealing the discontinuous structure arising from equilibrium regime switches at ω_H and ω_L.

4.3.1 Skill, Opacity, and Fragility

Proposition 4.1 and Figure 8 show that the manager's optimal opacity ω_η^* is globally increasing in average skill ω_s. The mechanism follows directly from the commitment role of opacity established in Corollary 4.1. A manager with higher ω_s expects to obtain a larger informational advantage and thus stands to gain more from an improvement in her profit margin. Since opacity mitigates the showing-off motive and steers the manager's strategy toward the profit-maximizing benchmark, the marginal benefit of obfuscation rises with ω_s. This positive skill–opacity relationship is consistent with the empirical pattern in which skilled managers disproportionately migrate to opaque investment vehicles and subsequently deliver superior returns (Kostovetsky, 2017).

Furthermore, the interaction between skill, obfuscation, and equilibrium selection generates a non-monotone relationship between opacity and financial fragility. As noted in Section 3, the equilibrium type is determined by the manager's opacity choice: transparent funds ($\omega_\eta < \omega_L$) support only the



(A) Low Average Skill (B) Intermediate Average Skill (C) High Average Skill

Figure 8: Average Skill and Optimal Obfuscation

Note: The figure plots the optimal degree of fund opacity ω_η^* against the manager's expected (average) skill ω_s, focusing on three regions that correspond to the equilibrium types. The dashed horizontal lines represent opacity thresholds, ω_L and ω_H, defined in Proposition 3.1. We use the parameter values $\phi = 0.03, n = 5, \omega_u = 5, \rho_H = 0.10$, and the obfuscation cost is $C = c\omega_\eta^\gamma$ with $c = 10^{-4}$ and $\gamma = 1.85$. For certain parameter configurations, the increasing region above ω_H is absent, and optimal opacity is pinned at ω_H for all sufficiently high skill levels. The qualitative features of the solution are nonetheless robust across a range of parameter values.

high-β equilibrium, funds with intermediate opacity ($\omega_L \leq \omega_\eta \leq \omega_H$) support multiple equilibria, and opaque funds ($\omega_\eta > \omega_H$) yield only the low-β equilibrium. Because optimal opacity is increasing in skill, this induces a systematic mapping from skill to fragility, such that managers with intermediate skill optimally choose intermediate opacity in $\omega_\eta \in [\omega_L, \omega_H]$, placing them in the multiple-equilibria region and making them a source of financial fragility. In contrast, high-skill managers operating under high opacity secure the unique low-β equilibrium with superior trading performance. Low-skill managers, for whom the marginal benefit of obfuscation is small, optimally choose low opacity and remain in the unique flow-driven equilibrium, avoiding fragility but at the cost of poor trading performance. While opacity is commonly viewed as a driver of financial instability, our model delivers a sharply different message: the most opaque funds are not the source of fragility; it is funds with *intermediate* opacity, operated by managers of intermediate skill, that contribute to systemic fragility. This result is broadly consistent with the observation in Zuckerman (2019) that certain highly opaque funds maintained stable, profitable operations during the 2008 financial crisis, while many other funds experienced severe instability.

This non-monotone relationship also carries a direct policy implication. A regulatory mandate that raises the cost of obfuscation—for example, through stricter disclosure requirements—shifts the optimal-opacity schedule downward. Managers who previously operated in the unique low-β region may find it no longer worth incurring the higher cost of obfuscation, and optimally reduce their opacity below ω_H, entering the fragile multiple-equilibria region. In other words, a transparency mandate can inadvertently push skilled, performance-driven managers into the very region of fragility it aims to suppress, meaning that transparency backfires.

4.3.2 Discontinuity and Constant Obfuscation

A salient feature of Figure 8 is that ω_η^* does not increase smoothly in ω_s, but instead exhibits two upward jumps, along with a region of constant opacity between them. The jumps arise because the manager's utility function is discontinuous at the equilibrium thresholds ω_L and ω_H. At $\omega_\eta = \omega_L$, the equilibrium transitions from unique high-β to multiple equilibria: the profitable low-β equilibrium may now be realized with probability ρ_L, generating a discrete upward jump in the manager's expected utility. The same logic applies at ω_H: entering the unique low-β region eliminates the possibility of inefficient high-β trading. Because the utility gain from crossing each threshold is discrete while the obfuscation cost is smooth, optimal opacity responds to marginal increases in ω_s with a discrete upward jump.

The numerical results further reveal a region in which ω_η^* is pinned at ω_H and is unresponsive to the expected skill. This is due to a trade-off that arises at the boundary of the unique low-β region: a manager in the multiple-equilibria region can capture a discrete utility gain by pushing opacity exactly to ω_H, eliminating the probability of high-β trading. Once at ω_H, however, the associated marginal benefit of additional obfuscation may not outweigh its marginal cost $C'(\omega_\eta)$. For managers whose skill is not high enough to justify this incremental cost, optimal opacity is pinned at ω_H. As skill rises sufficiently, the marginal benefit of opacity eventually dominates, and ω_η^* begins increasing again above ω_H.

These features carry concrete implications for the testable relationship between skill and opacity. First, while opacity and skill are positively correlated in general, the cross-sectional distribution of fund opacity should exhibit discrete mass points (i.e., clusters of funds) around the thresholds for multiple equilibria, rather than following a smooth unimodal distribution. Second, the constant-opacity region implies that skill is uninformative about opacity for a non-trivial range of managers: funds with meaningfully different skill levels optimally choose identical opacity levels. Within this region, however, fund performance should still be positively correlated with skill even after controlling for opacity, providing a testable prediction that disentangles the effects of skill and opacity.

5 Conclusion

This paper studies a model of delegated asset management in which a fund manager, privately informed about her skill, trades a risky asset while strategically designing the readability of fund communications. The model yields two stable, self-fulfilling equilibria: a Kyle-like equilibrium, in which the manager pursues profit-maximizing strategies, and a flow-driven equilibrium, in which she trades more aggressively to signal skill and attract capital, resulting in higher price volatility and lower trading profits. A mere shift in investor beliefs, unaccompanied by any change in fundamentals, can trigger a transition between equilibria, which we interpret as financial *fragility*.

We endogenize fund opacity and show that it serves as a commitment device: by obfuscating fund information, the manager curbs her own future showing-off incentive and steers herself toward profit-driven trading. Since this commitment value is greater for higher-skilled managers, optimal opacity increases with skill. This generates a non-monotone opacity–fragility relationship: intermediate-skill managers choose intermediate opacity, placing them in the fragile multiple-equilibria region, while high-skill managers operate stably under high opacity. A direct policy implication follows: transparency mandates can inadvertently push skilled managers into the fragile region, meaning that transparency backfires.

An interesting direction for future research would be to examine how competition among fund managers for overlapping investor bases affects obfuscation incentives, potentially generating strategic complementarities in opacity choices with implications for market-wide fragility.

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Online Appendix

A Proof of Lemma ??

From (3.1) and (3.2), p is observationally equivalent to $\frac{p-\hat{\delta}}{\lambda} = \beta s + u \equiv z_p$, which is the sum of two normally distributed variables from the market maker's perspective. Since $\delta \sim N(\hat{\delta} + s, \omega_\delta)$ is equivalently presented as $\delta = \hat{\delta} + s + \epsilon$ with $\epsilon \sim N(0, \omega_\delta)$, δ is observationally equivalent to $\delta - \hat{\delta} = s + \epsilon \equiv z_\delta$, which is also the sum of normal variables. Thus, given $Z = [z_p,\ z_\delta]^{\mathrm{T}}$, the market maker's posterior mean of s is

$$\mathrm{E}\,[s|Z] = \underbrace{\mathrm{E}[s]}_{=0} + \Sigma_{sZ}\Sigma_Z^{-1}(Z - \underbrace{E[Z]}_{=[0\ \ 0]^{\mathrm{T}}}), \tag{A.1}$$

where

$$\Sigma_{sZ} = \begin{bmatrix} \beta\omega_s, & \omega_s \end{bmatrix} \text{ and } \Sigma_Z = \begin{bmatrix} \beta^2\omega_s + \omega_u, & \beta\omega_s \\ \beta\omega_s, & \omega_s + \omega_\delta \end{bmatrix}. \tag{A.2}$$

Since

$$\begin{aligned} \det[\Sigma_Z] &= (\beta^2\omega_s + \omega_u)(\omega_s + \omega_\delta) - \beta^2\omega_s^2 \\ &= \omega_s(\beta^2\omega_\delta + \omega_u) + \omega_u\omega_\delta, \end{aligned} \tag{A.3}$$

it holds that

$$\begin{aligned} \mathrm{E}\,[s|Z] &= \begin{bmatrix} \beta\omega_s, & \omega_s \end{bmatrix} \frac{1}{\det[\Sigma_Z]} \begin{bmatrix} \omega_s + \omega_\delta, & -\beta\omega_s \\ -\beta\omega_s, & \beta^2\omega_s + \omega_u \end{bmatrix} \begin{bmatrix} z_p \\ z_\delta \end{bmatrix} \\ &= \frac{\omega_s}{\omega_s(\beta^2\omega_\delta + \omega_u) + \omega_u\omega_\delta} \left(\beta\omega_\delta z_p + \omega_u z_\delta\right) \\ &= \frac{\omega_s(\beta\omega_\delta + \omega_u)}{\omega_s(\beta^2\omega_\delta + \omega_u) + \omega_u\omega_\delta} \left(\frac{\beta\omega_\delta}{\beta\omega_\delta + \omega_u} z_p + \left(1 - \frac{\beta\omega_\delta}{\beta\omega_\delta + \omega_u}\right) z_\delta \right) \\ &= \xi\left(\gamma z_p + (1-\gamma) z_\delta\right), \end{aligned} \tag{A.4}$$

where

$$\xi \equiv \frac{\omega_s(\beta\omega_\delta + \omega_u)}{\omega_s(\beta^2\omega_\delta + \omega_u) + \omega_u\omega_\delta} \tag{A.5}$$

and

$$\gamma \equiv \frac{\beta\omega_\delta}{\beta\omega_\delta + \omega_u}. \tag{A.6}$$

Re-writing z_p and z_δ in (A.4) in terms of p and δ, we obtain the required result. $\square$

B Proof of Lemma ??

Let us present $\delta \sim N(\hat{\delta} + s, \omega_\delta)$ equivalently as $\delta = \hat{\delta} + s + \epsilon$ with $\epsilon \sim N(0, \omega_\delta)$. Then, squaring (**??**), we have

$$\begin{aligned} \hat{s}^2 &= \xi^2 \left(\gamma^2 x^2 + 2\gamma x \left(\gamma u + (1-\gamma)(\delta - \hat{\delta}) \right) + \left(\gamma u + (1-\gamma)(\delta - \hat{\delta}) \right)^2 \right) \\ &= \xi^2 \left(\begin{array}{c} \gamma^2 x^2 + 2\gamma^2 x u + 2\gamma(1-\gamma)(s+\epsilon)x \\ +\gamma^2 u^2 + 2\gamma(1-\gamma)(s+\epsilon)u + (1-\gamma)^2(s+\epsilon)^2 \end{array} \right). \end{aligned} \tag{B.1}$$

Taking the expectation of (B.1) from the trader's perspective,

$$\mathrm{E}[\hat{s}^2|s] = \xi^2 \left(\gamma^2 x^2 + 2\gamma(1-\gamma)sx + A \right), \tag{B.2}$$

where

$$A \equiv \gamma^2\omega_u + (1-\gamma)^2(s^2 + \omega_\delta) \tag{B.3}$$

is the term that the trader is unable to influence by her choice of x. We can omit A in the maximization problem below. Using (B.2), the trader's problem (**??**) is rewritten as

$$\begin{aligned} &\max_x \; \mathrm{E}\left[(\delta - p)x|s\right] + \theta\mathrm{E}[\hat{s}^2|s] \\ \Rightarrow \; &\max_x \; \mathrm{E}\left[\left(\hat{\delta} + s + \epsilon - \left(\hat{\delta} + \lambda(x+u) \right) \right) x \middle| s \right] + \theta\mathrm{E}[\hat{s}^2|s] \\ \Rightarrow \; &\max_x \; sx - \lambda x^2 + \theta\xi^2 \left(\gamma^2 x^2 + 2\gamma(1-\gamma)sx \right). \quad \square \end{aligned} \tag{B.4}$$

C Proof of Lemma ??

At the beginning of $t = 1$, the trader does not have reputation concerns anymore. Her problem is just to choose x_1 to maximize the expected trading profit, that is,

$$\max_{x_1 \in (-\infty, \infty)} \mathrm{E}[(\delta_1 - p_1) x_1 | s]. \tag{C.1}$$

We conjecture and later verify that the trader's equilibrium order in $t = 1$ is

$$x_1 = \beta_1 s, \tag{C.2}$$

where $\beta_1 > 0$ is a constant to be determined. Given (C.2), the market maker sets the price

$$\begin{aligned}
p_1 =& \mathrm{E}\left[\delta_1 | x_1 + u_1\right] \\
=& \mathrm{E}[\mathrm{E}[\delta_1 | x_1 + u_1, s] | x_1 + u_1] \\
=& \mathrm{E}[\hat{\delta}_1 + s | x_1 + u_1] \\
=& \hat{\delta}_1 + \frac{\mathrm{Cov}[s, x_1 + u_1]}{\mathrm{Var}[x_1 + u_1]} \left(x_1 + u_1 - \mathrm{E}[x_1 + u_1]\right) \\
=& \hat{\delta}_1 + \frac{\mathrm{Cov}[s, \beta_1 s + u_1]}{\mathrm{Var}[\beta_1 s + u_1]} \left(x_1 + u_1 - \mathrm{E}[\beta_1 s + u_1]\right) \\
=& \hat{\delta}_1 + \lambda_1 \left(x_1 + u_1\right),
\end{aligned} \tag{C.3}$$

where

$$\lambda_1 \equiv \frac{\beta_1 \omega_s}{\beta_1^2 \omega_s + \omega_{u,1}}. \tag{C.4}$$

Given (C.3), the trader's problem (C.1) is rewritten as

$$\max_{x_1} \ \mathrm{E}\left[\left(\delta_1 - \left(\hat{\delta}_1 + \lambda_1 (x_1 + u_1)\right)\right) x_1 \middle| s\right] \ \Rightarrow \ \max_{x_1} \ s x_1 - \lambda_1 x_1^2. \tag{C.5}$$

The FOC for x_1 is $s - 2\lambda_1 x_1 = 0$, which determines the trader's optimal action:

$$x_1 = \frac{1}{2\lambda_1} s. \tag{C.6}$$

For the other agents' belief (C.2) to be consistent with (C.6), we need

$$\beta_1 = \frac{1}{2\lambda_1} \iff \beta_1 = \sqrt{\frac{\omega_{u,1}}{\omega_s}}, \tag{C.7}$$

which implies

$$\lambda_1 = \frac{1}{2}\sqrt{\frac{\omega_s}{\omega_{u,1}}}. \quad \square \tag{C.8}$$

D Proof of Lemma ??

Let us present $\delta_1 \sim N(\hat{\delta}_1 + s, \omega_\delta)$ equivalently as $\delta_1 = \hat{\delta}_1 + s + \epsilon_1$ with $\epsilon_1 \sim N(0, \omega_\delta)$. Then the trader's trading profit in $t = 1$ is

$$\begin{aligned}
\pi_1 &= (\delta_1 - p_1)x_1 \\
&= \left(\hat{\delta}_1 + s + \epsilon_1 - \left(\hat{\delta}_1 + \lambda_1 (\beta_1 s + u_1)\right)\right)\beta_1 s \\
&= \left(\hat{\delta}_1 + s + \epsilon_1 - \left(\hat{\delta}_1 + \frac{1}{2}\sqrt{\frac{\omega_s}{\omega_{u,1}}}\left(\sqrt{\frac{\omega_{u,1}}{\omega_s}} s + u_1\right)\right)\right)\sqrt{\frac{\omega_{u,1}}{\omega_s}} s \\
&= \frac{1}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}} s^2 + \left(\sqrt{\frac{\omega_{u,1}}{\omega_s}}\epsilon_1 - \frac{u_1}{2}\right) s. \qquad \text{(D.1)}
\end{aligned}$$

Taking the expectation of (D.1) from the recruiter's perspective, we obtain the trader's salary:

$$\begin{aligned}
w &= \mathrm{E}[\pi_1 | p_0, \delta_0] \\
&= \frac{1}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}} \mathrm{E}[s^2 | p_0, \delta_0] \\
&= \frac{1}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}} \left(\mathrm{E}[s|p_0, \delta_0]^2 + \mathrm{Var}[s|p_0, \delta_0]\right) \\
&= \frac{1}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}} \left(\hat{s}^2 + \hat{\omega}_s\right), \qquad \text{(D.2)}
\end{aligned}$$

where $\hat{\omega}_s$ is the posterior variance of s, which is a constant. $\square$

E Proof of Propositions 3.1 and ??

In the following analyses, denote

$$a = \frac{\omega_{u,0}}{\omega_s},\ b = \frac{\omega_{u,0}}{\omega_\delta}, \tag{E.1}$$

so that we rewrite equation (3.15) as

$$\beta = F(\beta) \equiv \frac{1 + 2\theta \left(\frac{1}{a+b+\beta^2}\right)^2 b\beta}{2\left(\frac{\beta}{\beta^2+a} - \theta\left(\frac{\beta}{a+b+\beta^2}\right)^2\right)}. \tag{E.2}$$

By rearranging (E.2), the equilibrium β solves the following condition.

$$2\theta\beta = J(\beta^2), \tag{E.3}$$

where, by denoting $\beta^2 = y$,

$$J(y) = \frac{(y-a)\,(y+b+a)^2}{(y+a)\,(y+b)}. \tag{E.4}$$

The first-order derivative of the RHS of (E.3) with respect to β is

$$K^{(1)}(\beta) \equiv 2\beta J^{(1)}(\beta^2), \tag{E.5}$$

with

$$J^{(1)}(y) \equiv \frac{N(y)}{(y+a)^2\,(y+b)^2}, \tag{E.6}$$

where the numerator is

$$N(y) = y^4 + 2(a+b)y^3 + n_2 y^2 + n_1 y + n_0$$

with coefficients being

$$n_2 = 2a^2 + 6ab + b^2,$$

$$n_1 = 2a\left[3b^2 + 3ab + a^2\right],$$

$$n_0 = a\left(a^2 + ab + 2b^2\right)(a+b).$$

From (E.4) and (E.6), $y = a$ is the unique solution to $J(y) = 0$ in $y > 0$, and $J(y)$ is increasing in y for $y \geq a$. Hence, there is no equilibrium in $y < a$, and we focus on $y \geq a$.

The second-order derivative of the RHS of (E.3) with respect to β is expressed as

$$K^{(2)}(y) \equiv 2\left(J^{(1)}(y) + 2yJ^{(2)}(y)\right), \tag{E.7}$$

where $J^{(2)}$ denotes the second-order derivative of J with respect to y. $K^{(1)}$ and $K^{(2)}$ have the following properties:

Lemma E.1. *(i) $K^{(1)}(\beta)$ takes a U-shaped curve in relation to β.*
(ii) $K^{(2)}(a) < 0$ and $K^{(2)}(\infty) > 0$. There exists a unique solution to $K^{(2)}(y) = 0$.

Proof. Note that point (i) directly follows from point (ii). By denoting that $N^{(1)}(y) = \frac{dN(y)}{dy}$, it holds that

$$J^{(1)}(y) + 2yJ^{(2)}(y) = J^{(1)}(y)\frac{-7y^2 - 3(a+b)y + ab}{(y+a)(y+b)} + 2y\frac{\left(y^2 + (a+b)y + ab\right)N^{(1)}(y)}{(y+a)^3(y+b)^3}. \tag{E.8}$$

Evaluating at $y = a$,

$$\begin{aligned}
J^{(1)}(a) + 2aJ^{(2)}(a) &= -J^{(1)}(a)\frac{5a+b}{a+b} + \frac{N^{(1)}(y)}{2a(a+b)^2} \\
&\propto -2a(a+b)(5a+b)J^{(1)}(a) + N^{(1)}(a) \\
&= -4a^3 - b(4+b)a - b^3 \\
&< 0.
\end{aligned}$$

Therefore, $K^{(2)}(a) < 0$ holds.

The numerator of (E.8) is summarized as

$$T(y) = -\left[y^4 + 2(a+b)y^3 + n_2 y^2 + n_1 y + n_0\right]\left[7y^2 + 3(a+b)y - ab\right] + 2y\left(y^2 + (a+b)y + ab\right)\left[4y^3 + 6(a+b)y^2 + 2n_2 y + n_1\right].$$

The above argument suggests that $T(a) < 0$.

Using Mathematica, we summarize T into the following polynomial:

$$T(y) = \sum_{k=0}^{6} t_k y^k, \tag{E.9}$$

where the coefficients are

$$t_0 = a^2 b(a+b)\left(a^2 + ab + 2b^2\right) > 0,$$

$$t_1 = -3a\left(a^4 + (a+b)(a-b)^2 b + b^4\right) < 0,$$

$$t_2 = -3a\left(3a^3 + 4a^2 b + ab^2 + 5b^3\right) < 0,$$

$$t_3 = -8a^3 - 8a^2 b - 9ab^2 + b^3,$$

$$t_4 = 3b\,(a+b) > 0,$$

$$t_5 = 3(a+b) > 0,$$

and $t_6 = 1$. As $t_6 > 0$, $T(\infty) = \infty > 0$. Together with $T(a) < 0$, the intermediate value theorem implies that $T(y) = 0$ has at least one solution in $y > a$.

Suppose that y_{min} is one of the solutions to $T(y) = 0$. Consider the first-order derivative of T evaluated at $y = y_{min}$ and multiplied y_{min}:

$$\mathcal{T}(y_{min}) = y_{min}\frac{dT(y_{min})}{dy} = \sum_{k=1}^{6} k t_k y_{min}^k.$$

Applying $T(y_{min}) = 0$ and eliminating the terms related to t_3,

$$\mathcal{T}(y_{min}) = -3t_0 - 2t_1 y_{min} - t_2 y_{min}^2 + t_4 y_{min}^4 + 2t_5 y_{min}^5 + 3t_6 y_{min}^6,$$

According to the signs of t_k and $y_{min} > a$, it holds that

$$\begin{aligned}\mathcal{T}(y_{min}) &> -3t_0 - 2t_1 a - t_2 a^2 + t_4 a^4 + 2t_5 a^5 + 3t_6 a^6 \\ &= 3a^2\left(6a^4 + 8a^3 b + b^4 + (a^2 - b^2)^2\right) \\ &> 0.\end{aligned}$$

Therefore, if y_{min} is a solution to $T(y) = 0$, then the slope of $T(y)$ at this point must be positive. Together with $T(a) < 0 < T(\infty)$, y_{min} is the unique solution to $T(y) = 0$, and $T(y) > 0 \Leftrightarrow y > y_{min}$.[25] The argument above implies that $K^{(1)}(\beta)$ takes a U-shaped curve with respect to β, and the minimum point is given by $\beta_{min} \equiv \sqrt{y_{min}}$. Note that $\tilde{\beta}$ in footnote **??** is given by $\tilde{\beta} = \beta_{min}$. $\square$

The RHS of (E.3) is monotonically increasing in β, and Lemma E.1 attests that it is concave in $y \leq y_{min}$, while it becomes convex in $y > y_{min}$. In other words, the slope of the curve is the least steep at $y = y_{min}$, and it is given by

$$K_{min} = 2\beta_{min} J^{(1)}(y_{min}). \tag{E.10}$$

Note that K_{min} is represented only by parameters, a, b, and does not involve θ.

Consider the following condition to characterize equilibrium:

$$J(y_{min}) > K_{min}\beta_{min}. \tag{E.11}$$

Then the following result holds:

Lemma E.2. *(i) If inequality (E.11) holds, then there exist θ_H and θ_L with $\theta_H > \theta_L$. If, and only if, $\theta \in \Theta \equiv (\theta_L, \theta_H)$, then the equilibrium condition (E.3) has three solutions.*
(ii) If inequality (E.11) is violated, then $\theta_H = \theta_L$, and the equilibrium condition (E.3) has a unique solution. The solution is $\beta > \beta_{min}$ ($\beta < \beta_{min}$) if $\theta > \theta_H$

[25] Suppose that $T(y) = 0$ has more than two solutions. For any solutions, denoted as y_0 and y_1, the mean-value theorem implies that there exists at least one $y_2 \in [y_0, y_1]$ such that $\frac{dT(y_2)}{dy} = 0$. This contradicts to the strict positivity or negativity of $\mathcal{T}$.

($\theta < \theta_H$).

Proof. Consider the plot of the curve, $J(\beta^2)$, and the line, $2\theta\beta$, against β. With the curve being fixed, consider a line that goes through the origin and is tangent to the curve. The tangent point is characterized by β that satisfies

$$J(\beta^2) = K^{(1)}(\beta^2)\beta,$$

where the LHS represents the y-coordinate of the curve at β, and the RHS represents that of the line that goes through the origin. Note that their slopes must be identical and $K^{(1)}(\beta^2)$. Applying (E.5), the above condition is rewritten as

$$J(\beta^2) = 2\beta^2 J^{(1)}(\beta^2).$$

Hence, denoting $y = \beta^2$, the tangent point is characterized by the solution to

$$0 = G(y) = 2yJ^{(1)}(y) - J(y).$$

Note that we focus on $y > \sqrt{a}$, and thus y and β have one-to-one mapping.

Function $G(y)$ has the following first-order derivative:

$$\begin{aligned} G^{(1)}(y) &= 2\left(yJ^{(2)}(y) + J^{(1)}(y)\right) - J^{(1)}(y) \\ &= J^{(1)}(y) + 2yJ^{(2)}(y) \\ &= \frac{K^{(2)}(y)}{2}, \end{aligned}$$

where we apply the definition of $K^{(2)}$ in (E.7). Lemma E.1 has established that $K^{(2)}(a) < 0$ and $K^{(2)}(\infty) > 0$. Also, there is a unique $y = y_{min}$ such that $K^{(2)}(y_{min}) = 0$ and $K^{(2)}(y_{min}) > 0 \Leftrightarrow y > y_{min}$. Hence, $G(y)$ takes a U-shaped curve with respect to y and $y = y_{min}$ is the minimizer of the curve.

With $G(y)$ being U-shaped, $G(y) = 0$ has two solutions if and only if its minimum point dips below the x-axis, namely,

$$2y_{min}J^{(1)}(y_{min}) - J(y_{min}) < 0.$$

Applying the definition of K_{min} in (E.10), the above inequality is rewritten as condition (E.11).

When condition (E.11) holds and $G(y) = 0$ has two solutions, we denote them as y_0 and $y_1(> y_0)$. The corresponding slopes of these tangent lines are $K^{(1)}(y_0)$ and $K^{(1)}(y_1)$, respectively. Then the original line, $2\theta\beta$, and the curve, $J(\beta^2)$, have three intersections if and only if

$$\frac{K^{(1)}(y_1)}{2} \equiv \theta_L < \theta < \theta_H \equiv \frac{K^{(1)}(y_0)}{2}. \tag{E.12}$$

This result concludes the proof of statement (i) of the lemma.

Conversely, if condition (E.11) does not hold, no tangent line that goes through the origin is obtained: function $J(\beta^2)$ becomes the least steep at $\beta_{min} = \sqrt{y_{min}}$, and the tangent line is specified by $K_{min}\beta - K_{min}\beta_{min} + J(\beta_{min}^2)$, while the intercept of this line is negative, as condition (E.11) is violated. Hence, the line, $2\theta\beta$, and the curve, $J(\beta^2)$ have a unique intersection. This intersection occurs in $\beta > \beta_{min}$ if, and only if, $\theta > \theta'_H \equiv \frac{J(\beta_{min}^2)}{2\beta_{min}}$.

To conclude statement (ii), we must show that θ_H and θ_L in statement (i) converges to θ'_H in statement (ii) at the threshold of these two cases so that we re-define it as $\theta_H = \theta_L = \theta'_H$. This result is shown in Lemma E.3 below. $\square$

Finally, consider the comparative statics with respect to ω_s. The following result holds:

Lemma E.3. *(i) $J(y_{min}) - K_{min}\beta_{min}$ is increasing in ω_s. Hence, there exists a unique $\tilde{\omega}_s > 0$, such that, (E.11) holds with equality, and inequality (E.11) holds if and only if $\omega_s > \tilde{\omega}_s$.*
(ii) When condition (E.11) holds, both θ_H and θ_L increase as ω_s increases. Interval Θ expands, as the following inequalities hold:

$$0 < \frac{d\theta_L}{d\omega_s} < \frac{d\theta_H}{d\omega_s}.$$

(iii) At $\omega_s = \tilde{\omega}_s$, it holds that $\theta_H = \theta_L = \theta'_H$, while at the limit of $\omega_s \to 0$, $\theta'_H \to 0$.

(iv) There exists a unique threshold for φ^*, such that, $\varphi \geq \varphi^*(< \varphi^*)$ leads to case 1 (case 2) in Proposition **??**.

Proof of Statements (i) and (ii). Note that we are interested in the reaction of the following function when $a = \frac{\omega_{u,0}}{\omega_s}$ changes and, hereafter, we make the dependence of functions on a explicit:

$$2\theta_k = K^{(1)}(y_j, a) = 2\sqrt{y_j} J^{(1)}(y_j, a),$$

where $y_j = y_0, y_1$ denote the solutions to $G(y, a) = 0$, if any, and $k = L, H$ are appropriately determined following (E.12). According to the definition of G, the following partial derivative holds.

$$\frac{\partial G(y,a)}{\partial a} = 2y \frac{\partial J^{(1)}(y,a)}{\partial a} - \frac{\partial J(y,a)}{\partial a}.$$

The first term is summarized by

$$\begin{aligned}
\frac{\partial J^{(1)}(y,a)}{\partial a} &\propto (y+a)\frac{\partial N(y,a)}{\partial a} - 2N(y,a) \\
&= (y+a)\left[2y^3 + n_{2,a}y^2 + n_{1,a}y + n_{0,a}\right] - \left[y^4 + 2(a+b)y^3 + n_2 y^2 + n_1 y + n_0\right] \\
&= \sum_{k=0}^{4} A_k y^k
\end{aligned}$$

where $A_4 = 1,\ A_3 = 4(a+b)$,

$$A_2 = 2a^2 + 5b^2 + 2(3ab + a^2) + 2a\left[3b + 2a\right],$$

$$A_1 = 2a^2\left[3b + 2a\right]\left(a^2 + ab + 2b^2\right)(2a+b) + a\,(2a+b)\,(a+b),$$

$$\begin{aligned}
A_0 &= a\left[(2a+b)\left(a^2 + ab + 2b^2 + a(a+b)\right) - \left(a^2 + ab + 2b^2\right)(a+b)\right] \\
&> \left(a(a+b)\right)^2.
\end{aligned}$$

Hence, $\frac{\partial J^{(1)}(y,a)}{\partial a} > 0$. The second term is

$$(y+b)\frac{\partial J(y,a)}{\partial a} = -2\frac{(y+b+a)\left(y(a+b)+a^2\right)}{(y+a)^2} < 0. \tag{E.13}$$

Therefore, $\frac{\partial G}{\partial a} > 0$. As the previous proof has established that $G(y,a)$ takes a U-shaped curve, $\frac{\partial G}{\partial a} > 0$ implies that its minimum point is monotonically decreasing in ω_s, concluding the first statement of point (i) in the lemma.

As $a \to 0$, it holds that $y_{min} \to 0$ and

$$\lim_{a\to 0} G(y_{min}, a) = \lim_{a\to 0}[2y_{min} - (y_{min} + b)] = -b < 0.$$

Also

$$\lim_{a\to\infty} G(y_{min}, a) = +\infty.$$

Therefore, together with

$$\frac{dG(y_{min}, a)}{da} = \frac{dy_{min}}{da}\frac{\partial G}{\partial y} + \frac{\partial G}{\partial a} > 0,$$

the above limits imply the existence of $\bar{a}$, such that, condition (E.11) holds if and only if $a < \bar{a}$. Given $\omega_{u,0}$, this result can be translated into ω_s so that $\tilde{\omega}_s = \frac{\omega_{u,0}}{\bar{a}}$. Noting that $a < \bar{a} \Leftrightarrow \omega_s > \tilde{\omega}_s$, we conclude the second half of point (i) in the lemma.

Suppose that condition (E.11) holds, meaning that y_0 and y_1 exist. For $j \in 0, 1$, $\frac{\partial G}{\partial a} > 0$ implies that $\frac{dy_0}{da} > 0$ and $\frac{dy_1}{da} < 0$. Also, the implicit function theorem leads to

$$\frac{dy_j}{da} = -\frac{1}{G^{(1)}(y_j, a)}\frac{\partial G(y_j, a)}{\partial a}.$$

By taking the total derivative of θ_k,

$$\begin{aligned}\frac{d\theta_k}{da} &= \frac{1}{2}\frac{1}{\sqrt{y_j}}\frac{dy_j}{da}G^{(1)}(y_j,a) + \sqrt{y_j}\frac{\partial J^{(1)}(y_j,a)}{\partial a} \\ &= -\frac{1}{2}\frac{1}{\sqrt{y_j}}\left(2y_j\frac{\partial J^{(1)}(y_j,a)}{\partial a} - \frac{\partial J(y_j,a)}{\partial a}\right) + \sqrt{y_j}\frac{\partial J^{(1)}(y_j,a)}{\partial a} \\ &= \frac{1}{2}\frac{1}{\sqrt{y_j}}\frac{\partial J(y_j,a)}{\partial a} \qquad \text{(E.14)} \\ &< 0,\end{aligned}$$

where the last inequality follows from (E.13). Therefore, we conclude that the boundary slopes, θ_H and θ_L are both increasing in ω_s.

Consider the magnitude of the reactions. By plugging (E.13) into (E.14),

$$\frac{d\theta_k}{da}| \propto \frac{\sqrt{y_j}}{(y_j+a)^2}\left(1+\frac{a}{y_j+b}\right)\left(1+\frac{1}{y_j}\frac{a^2}{a+b}\right).$$

It is straightforward that the last terms are all decreasing in y_j. Since we compare $y_0 < y_1$ with a being fixed, we obtain $\frac{d\theta_H}{d\omega_s} > \frac{d\theta_L}{d\omega_s}$, thereby concluding the proof.

Proof of Statements (iii) and (iv). Note that $\lim_{\omega_s \to \tilde{\omega}_s} y_j = y_{min}$, and $J(y_{min}) = K_{min}\beta_{min} = 2\beta_{min}J^{(1)}(y_{min})$ holds. Therefore, at this limit,

$$\theta'_H = \frac{J(\beta_{min}^2)}{2\beta_{min}} = \beta_{min}J^{(1)}(\beta_{min}^2) = \frac{K_{min}}{2} = \theta_k,$$

where the last equality holds because of (E.5). This concludes the proof of statement (iii).

Finally, consider the limit at $\omega_s \to 0$, which implies $a \to \infty$ (given $\omega_{u,0}$). Since $y_{min} \searrow a$, we can denote it as $y_{min} = a + \epsilon$, where $\epsilon > 0$ is a small positive and $\lim_{\omega_s \to \tilde{\omega}_s} \epsilon = 0$. With this notation, function J is rewritten at the limit as

$$\lim_{\omega_s \to \tilde{\omega}_s} J(y_{min}, a) = \lim_{a\to\infty,\epsilon\to 0}\frac{\epsilon(2a+b+\epsilon)}{(2a+\epsilon)(a+b+\epsilon)} = 0.$$

Since $\theta'_H = \frac{J(y_{min})}{2\sqrt{y_{min}}}$, we conclude that $\lim_{\omega_s \to \tilde{\omega}_s} \theta'_H = 0$.

As for statement (iv), case 1 occurs if, and only if, the curve $\theta = \frac{\varphi}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}}$ exceeds θ_H at $\omega_s = \tilde{\omega}_s$. The above analyses have already established that $\theta_H = \frac{K^{(1)(y_{min})}}{2}$, where y_{min} is implicitly characterized by parameters a and b, we can define the threshold φ^* as

$$\varphi^* \equiv K^{(1)(y_{min})}\sqrt{\frac{\omega_{u,0}}{\bar{a}\omega_{u,1}}}. \tag{E.15}$$

□

Note that Proposition 3.1 follows from points (i) and (ii) of Lemma E.3, whereas Proposition **??** is shown by putting monotonically decreasing θ in equation (**??**) together with points (i) and (ii).

F Proof of Lemma ?? and Proposition ??

The optimal trading strategy satisfies the following condition.

$$\therefore (1 - 2\lambda_0\beta_0) + \frac{\psi}{2\lambda_1}\xi^2(\gamma^2\beta_0 + \gamma(1-\gamma)) = 0. \tag{F.1}$$

The marginal benefit of increasing s^* is given by

$$\mathrm{E}_\beta[v(\omega_s, \beta_0)] = \left[(1 - \lambda_0\beta_0) + \frac{\psi}{4\lambda_1}\xi^2(\gamma^2\beta_0 + 2\gamma(1-\gamma))\right]\beta_0 + \frac{\psi}{4\lambda_1}\xi^2(1-\gamma)^2 + (1-\psi)\frac{1}{2}\sqrt{\frac{\omega_{u,1}}{\omega_s}}. \tag{F.2}$$

Then, it holds that

$$\begin{aligned}\frac{\partial \mathrm{E}_\beta[v(\omega_s, \beta_0)]}{\partial\beta_0} &= \left[(1 - 2\lambda_0\beta_0) + \frac{\psi}{2\lambda_1}\xi^2(\gamma^2\beta_0 + \gamma(1-\gamma))\right] \\ &+ \left[-\beta_0\frac{\partial\lambda_0}{\partial\beta_0} + \frac{\psi}{4\lambda_1}\frac{\partial}{\partial\beta_0}\left[\xi^2(\gamma^2\beta_0 + 2\gamma(1-\gamma))\right]\right]\beta_0 + \frac{\psi}{4\lambda_1}\frac{\partial}{\partial\beta_0}\left(\xi^2(1-\gamma)^2\right).\end{aligned}$$

Note that the first term is zero due to the envelope theorem using (F.1). By using the notations introduced in equations (E.1), the remaining terms are summarized as follows.

$$-\beta_0 \frac{\partial}{\partial \beta_0} \frac{\beta_0}{\beta_0^2 + a} + \frac{\psi}{4\lambda_1} \left[\beta_0^2 \frac{\partial}{\partial \beta_0} \left(\frac{\beta_0}{a + b + \beta_0^2} \right)^2 + 2\beta_0 \frac{\partial}{\partial \beta} \left(\frac{1}{a + b + \beta_0^2} \right)^2 b\beta_0 + \frac{\partial}{\partial \beta} \left(\frac{b}{a + b + \beta_0^2} \right)^2 \right]. \tag{F.3}$$

From the proof in Appendix E, it holds that $y > a$ in the equilibrium. Therefore, the first term is negative. As for the second term, it holds that

$$\begin{aligned} \text{Second terms of (F.3)} &= 2\beta_0^3 \left(\frac{a + b - \beta_0^2}{(a + b + \beta_0^2)^3} \right) + 2 \frac{a + b - 3\beta_0^2}{(a + b + \beta_0^2)^3} b\beta_0 - 4b^2 \frac{\beta_0}{(a + b + \beta_0^2)^3} \\ &\sim -y^2 + y(a + b - 3b) + b(a - b). \end{aligned}$$

Since $y > a$, the quadratic equation above is negative for all $y > a$ if and only if it is negative at $y = a$. This is true because

$$-a^2 + a(a - 2b) + b(a - b) = -ab - b^2 < 0.$$

Therefore, the marginal benefit in (F.2) is decreasing in β_0, concluding the proof of Lemma **??**. Proposition **??** directly follows from Lemma **??** and equation (**??**).