

FINA 4103
Class Note 3: Orders and Quotes

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Trading Needs

Why markets exist?

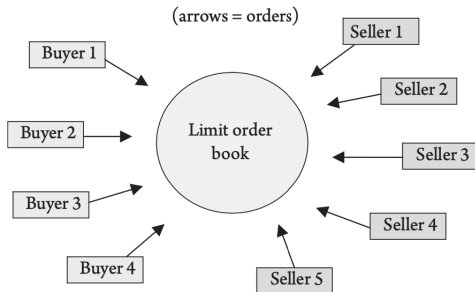
- ▶ To exchange values
- ▶ Various motivation to trade;
 - ▶ Private values (“I love Apple Inc. products”);
 - ▶ Information and belief (“I know good news about AAPL”);
 - ▶ Non-fundamental needs for trade (“I need cash, so sell AAPL”)
- ▶ How to express and execute your trading needs?
- ▶ How does your strategy depend on your motivation?

Trading Orders

- ▶ You place an **order** to express/fulfill your trading needs
 - ▶ Buying a box of wine via Amazon?
 - ▶ Check the price and place your “buy” order
- ▶ Buying Amazon stocks? — it's more complicated
- ▶ We will learn
 - ▶ what kind of orders are available;
 - ▶ how these orders are executed; and
 - ▶ US market structure and rules

Market Structure Recap

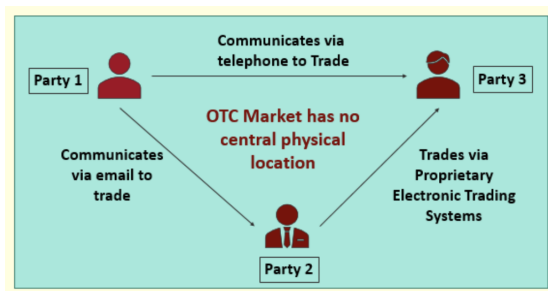
- ▶ Centralized limit-order market:
 - ▶ buyers and sellers send orders to a centralized “order book”
 - ▶ they are executed according to order types and LOB algorithm



Market Structure Recap

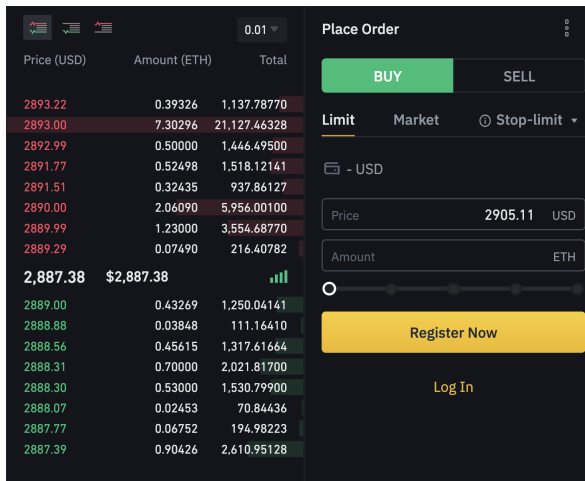
- ▶ Over-the-Counter:

- ▶ buyers and sellers are connected via decentralized network and trade P2P
- ▶ ECNs (LOB), direct communications, etc



- ▶ Our focus is on the centralized exchange with continuous LOB

Trading Orders: The Actual Order Book



Different exchanges offer different order types

NYSE Pillar Binary Gateway Order Type Matrix

Legend: For legend and guidelines on reading this document, refer to the "Order Types" Appendix of the

NYSE Pillar Gateway Binary Protocol Specification – iOptions Markets

54

NYSE Pillar Gateway Library Protocol Identification - IToolbox Markets

[illegible]

Available Order Types at IEX

Different exchanges offer different order types

Order Type Summary

Signal Series	Display Eligible	Reserve Eligible	Min. Quantity Eligible	Limit Price Required	Book Recheck Eligible
Discretionary Peg (D-Peg)	×	×	✓	Optional	✓
Primary Peg (P-Peg)	×	×	✓	Optional	✓
Corporate Discretionary Peg (C-Peg)	×	×	✓	Optional	✓
Discretionary Limit (D-Limit)	✓	✓	✓	✓	Displayed orders: No Non-displayed orders: Yes

Standard Series	Display Eligible	Reserve Eligible	Min. Quantity Eligible	Limit Price Required	Book Recheck Eligible
Market	×	×	✓	No, not accepted	✓
Limit	✓	✓	✓	✓	Displayed orders: No Non-displayed orders: Yes
Midpoint Peg (M-Peg)	×	×	✓	Optional	✓
Offset Peg (O-Peg)	×	×	✓	Optional	✓
Market Peg	×	×	✓	Optional	✓

Important Factors

- ▶ Price, quantity, and trading direction (buy/sell)
- ▶ Priority and speed
 - ▶ Execution quality and execution risk
 - ▶ If you are slow, you may miss the best price
- ▶ Transparency
 - ▶ Is your order publicly displayed?
- ▶ These factors are inter-dependent
 - ▶ If your willingness to pay is very high, immediately executed

Basic Orders

- ▶ **Limit** and **market** orders
 - ▶ price, quantity, and speed
- ▶ For both types, you need to specify
 - ▶ trading direction; “buy” or “sell”
 - ▶ trading quantity; how many units (shares) to trade
- ▶ They are different in terms of price qualifier

Limit Orders

- ▶ A buy limit order specifies the **bid price**
 - ▶ The maximum price at which you are prepared to buy a stated amount of security
- ▶ A sell limit order specifies the **ask price**
 - ▶ The minimum price at which you are prepared to sell a stated amount of security
- ▶ Limit orders may not find a counterpart
 - ▶ Unfilled limit orders are displayed on limit-order book and wait for transaction
 - ▶ For patient but price-sensitive traders (?)

Limit Orders: Example 1

- ▶ Start with an empty book
- ▶ If you place “limit sell for 100 shares at ask price \$100”...

Empty book

Sell (ask)		Buy (bid)
	\$101	
	\$100	
	\$99	
	\$98	
	\$97	



After you place your order

Sell (ask)		Buy (bid)
	\$101	
100	\$100	
	\$99	
	\$98	
	\$97	

Limit Orders: Example 2

- ▶ If your friend places “limit buy for 50 shares at price \$99”...

After your friend places order

Sell (ask)		Buy (bid)
	\$101	
100	\$100	
	\$99	
	\$98	
	\$97	



Sell (ask)		Buy (bid)
	\$101	
100	\$100	
	\$99	50
	\$98	
	\$97	

- ▶ What if the order was “limit buy for 50 shares at price \$101?”...

Market Orders

- ▶ **Market orders** do not specify the price;
 - ▶ Willing to trade whatever the price is
 - ▶ Can be seen as limit orders with $\text{ask} = -\infty$ and $\text{bid} = \infty$
- ▶ Filled immediately if there exists a standing limit order on the other side
- ▶ For price-insensitive but impatient traders

Market Orders

- If you send “buy market order of 150 shares”...

Sell (ask)		Buy (bid)
200	\$101	
80	\$100	
100	\$99	
	\$98	
	\$97	



After you place order

Sell (ask)		Buy (bid)
200	\$101	
30	\$100	
0	\$99	
	\$98	
	\$97	

Limit vs. Market Orders: Liquidity

- ▶ **Liquidity** $\approx$ how easy to trade
 - ▶ $\approx$ trading opportunities (or how costly to take them)
 - ▶ How market and limit orders differ in terms of liquidity?
- ▶ Market orders:
 - ▶ Your trading need is (almost) always fulfilled
- ▶ Limit orders:
 - ▶ Suggesting how much you are willing to trade at what price
 - ▶ Commitment to trade
- ▶ Collection of limit orders = the set of trading opportunities
 - ▶ standing limit orders are liquidity-provision orders
 - $\Leftrightarrow$ market orders are liquidity-taking orders

Limit vs. Market Orders: Liquidity

Limit Orders and Liquidity

Suppose that a limit buy order for 100 shares with bid price \$100 is placed on the book below. Is this limit order supplying or taking liquidity? What if the bid price is \$99?

Sell (ask)		Buy (bid)
100	\$101	
100	\$100	
40	\$99	
	\$98	
	\$97	



Incoming limit buy order?

Sell (ask)		Buy (bid)
	\$101	
	\$100	
	\$99	
	\$98	
	\$97	

Limit vs. Market Orders: Liquidity

- ▶ Collection of limit orders = the set of trading opportunities
 - ▶ standing limit orders are liquidity-provision orders
⇔ market orders are liquidity-taking orders
- ▶ Called “**marketable limit order**”
 - ▶ One with bid/ask price above/below the best available offers
 - ▶ (Partially) takes liquidity
- ▶ If fully executed, it is the same as market orders (from ex-post perspective)
 - ▶ Ex-ante, you still specify the price ($\neq$ market order)

Limit vs. Market Orders: Trade Quality

Execution risk

- ▶ They offer different execution risk and speed
- ▶ Limit orders; immediate execution is not guaranteed
 - ▶ may not be executed at all
- ▶ Market orders; executed immediately upon submission

Limit vs. Market Orders: Trade Quality

Execution price

- ▶ Market orders are likely to be executed at inferior prices
 - ▶ No control over the price
- ▶ Limit orders always trade at better (or equal) prices than quoted
- ▶ **Tradeoff** between execution speed and price

	Immediacy	Price
Market	✓	
Limit		✓

Limit vs. Limit Orders: Price-Time Priority

- ▶ **Price-time priority**

- ▶ The execution rule for multiple competing limit orders

- ▶ Ex., two limit sell orders for 100 shares are placed:

1. One with a better **price** (from buyers' POV) has higher priority
 - ▶ Placed on the top of the book
2. If they quote the same ask price, then arrival **time** is used as tie-breaker
 - ▶ First come, first served

- ▶ LOB must have such a priority rule $\Rightarrow$ a severe problem in modern market

Limit-Order Book Jargon



Best Bid and Offer (Ask)

- ▶ **Best bid** = the highest bid in the standing limit orders
- ▶ **Best ask (offer)** = the lowest ask in the standing limit orders
- ▶ Together they are “**Best Bid and Offer (BBO)**”
- ▶ BBO among all markets are “**National BBO (NBBO)**”

Exchange A

Sell (ask)		Buy (bid)
100	\$101	
100	\$100	
100	\$99	
	\$98	
	\$97	100

Exchange B

Sell (ask)		Buy (bid)
100	\$101	
100	\$100	
	\$99	
	\$98	100
	\$97	100

Bid-ask Spread and Mid-Point Quote

- ▶ **Bid-ask spread** = difference between best ask and best bid
 - ▶ In the multi-market environment, use NBBO (Class Note #4)
- ▶ **Mid-point quote (mid price)** = $\frac{\text{Best ask} + \text{Best bid}}{2}$
 - ▶ Typically used as a “fair” price of the asset (Class Note #7)

Exchange A

Sell (ask)		Buy (bid)
100	\$101	
100	\$100	
100	\$99	
	\$98	
	\$97	100

Exchange B

Sell (ask)		Buy (bid)
100	\$101	
100	\$100	
	\$99	
	\$98	100
	\$97	100

Bid-ask Spread and Liquidity

- ▶ Bid-ask spread is often used as a metric of liquidity
 - ▶ Liquidity: how easy to trade, cost of fulfilling trading needs
- ▶ You must pay the spread to conduct a *round-trip* trade
 - ▶ Small buy and sell (e.g., one unit)

Sell (ask)		Buy (bid)
100	\$101	
	\$100	
	\$99	100

- ▶ How is spread determined? What is its impact on trading behavior? ⇒ Second half of the course

Far Side, Near Side

- ▶ If you buy (resp. sell),
 - ▶ the best bid (resp. ask) is the **near side** of the spread
 - ▶ the best ask (resp. bid) is the **far side** of the spread

If you are selling

Sell (ask)		Buy (bid)
100	\$101	
100	\$100	
	\$99	
	\$98	100
	\$97	100

If you are buying

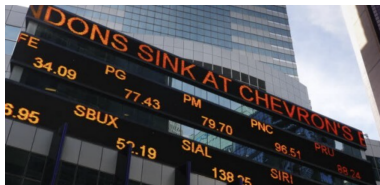
Sell (ask)		Buy (bid)
100	\$101	
100	\$100	
	\$99	
	\$98	100
	\$97	100

near side

far side

The “Price” of an Asset

- ▶ What are the prices shown on the “stock ticker tapes”?
- ▶ What is the price shown in the middle of the LOB?
- ▶ What do we mean by “AAPL is now \$100”?



- ▶ Typically, they mean the last execution price
≠ best bid/ask, mid-point quote

Market Crossing and Locking

- ▶ If “best bid = best ask”, the market is **locked**
 - ▶ If an order makes “best bid = best ask,” it locks the market
- ▶ If “best bid > best ask”, the market is **crossed**
 - ▶ an order trading beyond the far side price is crossing the market

Locked Market

Sell (ask)		Buy (bid)
100	\$101	
100	\$100	
100	\$99	50
	\$98	80
	\$97	100

An Order Crossing the Market

Sell (ask)		Buy (bid)
100	\$101	
100	\$100	
	\$99	
	\$98	100
	\$97	100

Limit buy of 300 shares at bid = \$102

Market Crossing and Locking

- ▶ Do these states even exist? Are they relevant?

Locked Market

Sell (ask)		Buy (bid)
100	\$101	
100	\$100	
100	\$99	50
	\$98	80
	\$97	100

An Order Crossing the Market

Sell (ask)		Buy (bid)
100	\$101	
100	\$100	
	\$99	
	\$98	100
	\$97	100

Limit buy of 300 shares at bid = \$102



- ▶ In a single-exchange environment, no.
- ▶ They are relevant when analyzing
 - ▶ multiple exchanges (e.g., the US); and
 - ▶ LOB data, which may exhibit crossed/locked market

More Sophisticated Order Types

Stop Orders

To better handle the tradeoff:

- ▶ **Stop orders**; specify the “trigger (stop)” price P_S
 - ▶ Can be both market and limit orders
- ▶ Instruction to buy (or sell) only if the last execution price goes beyond (below) P_S

Stop Orders: Example

- ▶ Stop limit sell for 100 shares at ask = \$113, stop price = \$115
 - ▶ It will be activated (placed) if $P \leq P_S$
 - ▶ As long as the price is sufficiently high, you hold your stock
 - ▶ Only if the price starts to decline, you sell your stock

As long as $P > \$115$

Sell (ask)		Buy (bid)
100	\$118	
100	\$117	
	\$116	80
	\$115	80
	\$114	100
	\$113	120
	\$112	110



At the timing of $P = \$115$

Sell (ask)		Buy (bid)
100	\$118	
100	\$117	
50	\$116	
20	\$115	
	\$114	10
	\$113	40
	\$112	90



Sell (ask)		Buy (bid)
100	\$118	
100	\$117	
50	\$116	
20	\$115	
	\$114	10
50	\$113	40
	\$112	90

Limit sell of 100 shares at \$113

Stop Orders: Example

Exercise

In the same situation, suppose that you set the ask price of your stop limit sell at \$117. What does the LOB in the last picture look like? What if your order is stop market sell?

As long as $P > \$115$

Sell (ask)		Buy (bid)
100	\$118	
100	\$117	
	\$116	80
	\$115	80
	\$114	100
	\$113	120
	\$112	110



At the timing of $P = \$115$

Sell (ask)		Buy (bid)
100	\$118	
100	\$117	
50	\$116	
20	\$115	
	\$114	10
	\$113	40
	\$112	90



Instruction to (not to) Cancel

Unexecuted limit orders stay on the book forever?

- ▶ **Good-till-cancel (GTC):**

- ▶ Your order is active on the LOB unless it is cancelled
- ▶ Trader or broker places cancellation request

- ▶ **Day orders:**

- ▶ Your order is automatically cancelled at the end of the trading session
- ▶ This is a typical (default) instruction for limit orders

- ▶ **Extended hours: specify more detailed time intervals**

- ▶ The pre-market (8:00–9:15)
- ▶ The after-market (16:15–18:30)

Open/Close Orders

- ▶ Orders with “**Open/Close**” instruction:
 - ▶ Market/limit on open orders: activated at the open
 - ▶ Market/limit on close orders activated at the close
- ▶ All these orders specify the timing (duration) of orders
- ▶ How to compute their time priority?

Instruction to Cancel: IOC

- ▶ **Immediate-or-Cancel (IOC)** orders
 - ▶ Cancel an order if it is not filled immediately
 - ▶ If partially filled, unfilled part is cancelled
- ▶ What happens to “IOC limit buy for 500 shares at \$50”?

Sell	Price	Buy
100	50.01	
200	50.00	500
	49.99	400
	49.98	200

Sell	Price	Buy
100	50.01	
200	50.00	500
	49.99	400
	49.98	200

Sell	Price	Buy
100	50.01	
	50.00	
	49.99	400
	49.98	200

- ▶ What does “immediate” mean?

Instruction to Cancel: IOC

- ▶ Why you add “IOC”?
 - ▶ Orders always take liquidity if not cancelled
 - ▶ Why don't you place a marketable limit and then manually cancel it?
- ▶ The share of IOC has dramatically increased in 2000's (Cao, et al, 2022, JFE)

IOC and High-Frequency Trading

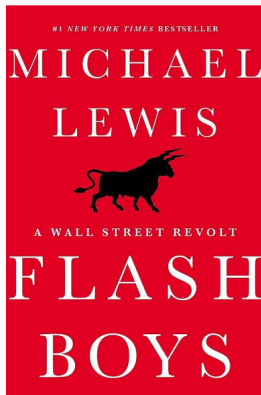
- ▶ You and your rival want to buy 100 units at \$115 immediately, but \$116 is too expensive
- ▶ Also, do not want to commit to trade later (e.g., due to price risk)
- ▶ You are equally fast; each order is executed first with prob $1/2$

Sell (ask)		Buy (bid)
100	\$118	
90	\$117	
50	\$116	
100	\$115	
	\$114	
	\$113	120
	\$112	110

- ▶ Market, limit, or IOC order?

Phantom Liquidity and High-Frequency Trading

- ▶ The above example is called “phantom liquidity”
 - ▶ A quote appears in LOB data feed
 - ▶ But when you place an order to take it, it disappears
- ▶ *Flash Boys* by Michael Lewis (2014)



Phantom Liquidity and High-Frequency Trading

- ▶ This happens due to difference in speed;
 - ▶ speed of data feed
 - ▶ speed of data processing
 - ▶ speed of reaction and order placement
- ▶ Example: NYSE provides two types of data access
 - ▶ **Direct data feed**; raw data of LOB, super fast
 - ▶ **Consolidated data feed**; processed aggregate data, slow
- ▶ Later, we will see details in “speed” dimension of information and trading

Instruction to Cancel Based on Quantity

Some other instructions similar to IOC

- ▶ **All-or-Nothing, All-or-None (AON):**
 - ▶ Orders cannot be partially filled
 - ▶ Your “limit sell for 100 units with AON” cannot be traded against “market buy of 50 units”
 - ▶ Held by your broker until sufficient liquidity shows up on the LOB
- ▶ **Fill-or-Kill (FOK):**
 - ▶ Combination of AON and IOC
 - ▶ If not fully filled immediately, cancelled

Price Qualifiers: Pegged Orders

Control the execution price by adding price qualifiers

- ▶ **Pegged orders** IEX
 - ▶ Your limit price dynamically changes
- ▶ Several types, e.g.,
 - ▶ **Primary peg:** your limit price is pegged at NBBO
 - ▶ **Midpoint peg:** your limit price is pegged at the midpoint of NBBO

Primary Peg Orders

Primary peg (P-peg) limit buy (resp. sell) order

1. Bid (ask) price is automatically set at one tick¹ below (above) the best bid (ask)
 2. Not displayed on the LOB (hidden order); and
 3. Trade up to the national best bid (ask)
- ▶ Limit price dynamically tracks the NBBO
 - ▶ Since it is hidden, lower priority to displayed orders in execution at the same price (see later)

¹Tick size is the minimum unit of the price grid, e.g., \$0.01

Example: Primary Peg Orders

- ▶ Suppose that NBO is \$10.04, while IEX LOB is like the left panel
- ▶ “P-peg limit sell order for 100 shares”

National Best Offer = \$10.04

Sell (ask)		Buy (bid)
90	\$10.06	
	\$10.05	
	\$10.04	
	\$10.03	
	\$10.02	
	\$10.01	
	\$10.00	120
	\$9.99	110

P-peg sell order of 100 shares

Sell (ask)		Buy (bid)
90	\$10.06	
100	\$10.05	
	\$10.04	
	\$10.03	
	\$10.02	
	\$10.01	
	\$10.00	120
	\$9.99	110

- 
- (1) Order is placed at one tick above the best ask
 - (2) P-peg order is not displayed on LOB

Example: Primary Peg Orders

- ▶ What happens when
 1. "limit buy for 100 shares at \$10.05" arrives
 2. "limit buy for 100 shares at \$10.04" arrives

Limit buy of 100 shares at \$10.05

Sell (ask)		Buy (bid)
90	\$10.06	
100	\$10.05	
	\$10.04	
	\$10.03	
	\$10.02	
	\$10.01	
	\$10.00	120
	\$9.99	110

Limit buy of 100 shares at \$10.04

Sell (ask)		Buy (bid)
90	\$10.06	
100	\$10.05	
100	\$10.04	
	\$10.03	
	\$10.02	
	\$10.01	
	\$10.00	120
	\$9.99	110

Example: Primary Peg Orders

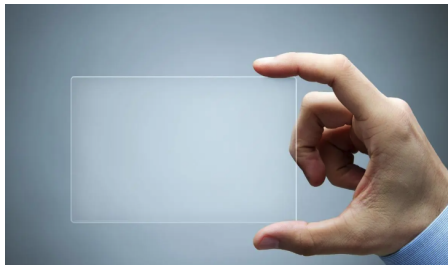
- ▶ When the National Best Offer changes to \$10.03

NBO goes down to \$10.03

Sell (ask)		Buy (bid)
90	\$10.06	
100	\$10.05	
100	\$10.04	
	\$10.03	
	\$10.02	
	\$10.01	
	\$10.00	120
	\$9.99	110

Transparency

- ▶ Limit orders and quotes show traders' intensity, perhaps revealing your private and material information
 - ▶ A large (block) buy order at a specific price means what?
- ▶ Display rules matter a lot



Hidden Orders

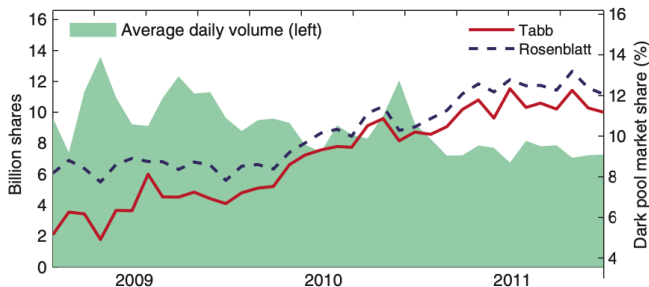
- ▶ Orders can be sent with a display instruction
- ▶ **Hidden orders, Reserve orders:**
 - ▶ Instruction to hide a part of trading amount
 - ▶ % of hidden portion is typically controlled by traders
 - ▶ Ex., 100 shares of sell hidden limit order at \$100 with 50% display
⇒ only 50 shares are displayed
- ▶ They have sizable shares
 - ▶ More than 18% in NASDAQ, 20% in Frankfurt, 22% Euronext-Paris (2019 data)
- ▶ Hidden orders have lower priority in execution
 - ▶ Displayed orders standing at the same price are executed first
 - ▶ Why lower priority?

Hidden Orders

- ▶ Dark side: they do not contribute to (or even harm) price discovery process
- ▶ Blight side: promote competition among liquidity providers and reduce trading cost for others
 - ▶ Boulatov and George (2013, RFS): hidden orders benefit the overall market
 - ▶ Zhu (2014, RFS): dark pool attracts more informed trading

Dark Pools

- ▶ Major ATS where all orders are completely hidden
- ▶ Trades are executed at the midpoint of NBBO
 - ▶ if $NBO = \$101$, $NBB = \$99$, orders sent to dark pools trade at \$100
 - ▶ So dark pools offer better prices
- ▶ Sizable shares; regulators concerned about price discovery



Zhu (2014) “Do Dark Pools Harm Price Discovery?”

- ▶ Consider two types of traders
 - ▶ Informed traders, trading on material private information
 - ▶ They contribute to price discovery
 - ▶ Noise traders, randomly buying and selling motivated by exogenous needs
- ▶ Asset value is either $v = v_H$ or v_L (not publicly observable)
- ▶ Informed traders obtain news about v
- ▶ Compare two cases;
 1. Regular “lit” exchange + Dark pool
 2. No dark pool (just a lit exchange)

Dark Pools Contribute to Price Efficiency

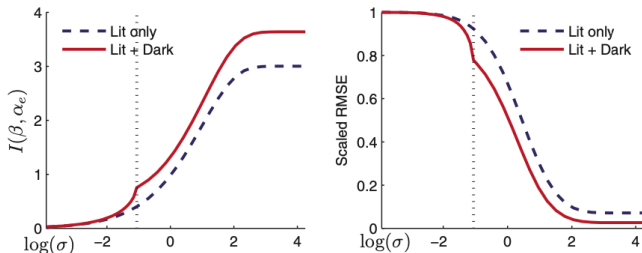


Figure 5

Two measures of price discovery, as functions of volatility σ

The left-hand plot shows the signal-to-noise ratio, and the right-hand plot shows scaled RMSE. The true asset value is $v = \sigma$, and other model parameters are those of Figure 3.

- ▶ Dark pool makes the price more efficient
 - ▶ I (left): price informativeness
 - ▶ RMSE (right): mean-squared error of the price compared to true asset value
- ▶ But why?—analyze how traders choose between dark and lit

Dark Pools Contribute to Price Efficiency

- ▶ Key: price improvement vs execution risk in dark pool
 - ▶ How is the risk defined?
 - ▶ How does this tradeoff differ among traders?
- ▶ Dark pool executes orders on *pro-rata* base
 - ▶ Called “crossing network”
- ▶ Let's say each trader buys or sells one unit:

$$\text{Prob}(\text{buy execution}) = \min \left\{ 1, \frac{\# \text{ of sellers}}{\# \text{ of buyers}} \right\}$$

- ▶ Informed and noise traders face different risk!

Dark Pools Contribute to Price Efficiency

- ▶ **Informed traders** face high execution risk
 - ▶ good news $\Rightarrow$ cluster on the buy side
- ▶ **Noise traders** are just randomly buying and selling (with prob 0.5)

$$E[\text{prob for informed}] = \min \left\{ 1, \frac{\text{noise sell}}{\text{noise buy} + \text{informed buy}} \right\}$$

Dark Pools Contribute to Price Efficiency

- ▶ **Informed traders** tend to use lit exchange, avoiding higher risk in dark pool
- ▶ **Noise traders** use dark pool to enjoy improved price
- ▶ Dark pool helps lit exchange “sort out” noise and cleanses it
⇒ information in lit exchange becomes more accurate
- ▶ Caveat: monopolistic informed trader may act differently

Next Class

- ▶ Multiple exchanges coexist, e.g., NYSE and NASDAQ
- ▶ A company can cross-list (dual-list) its stocks in multiple exchanges
 - ▶ Liquidity, information, international investor base
 - ▶ Ex., Walgreen, Microsoft, Intel,... are listed on NYSE and NASDAQ
- ▶ How to trade these stocks? Where? At what price?
 - ▶ We learn the US market structure