# FINA 3103: Intermediate Investment
# Class 3: Risk, Return, and Capital Allocation

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# Today

- The rate of return and risk of asset holding
  - Definition
  - How they vary across time and assets?

# Interest Rate

- **Interest rate**: $$ that a lender charges a borrower (as proportion of loan amount)
  - Borrow \$1,000 with 10% interest rate $\Rightarrow$ interest payment is \$100
- **Nominal and Real interest rates**

**Nominal and real interest rates**

If $i$ = nominal rate/ $r$ = real rate/ $\pi$ = inflation rate, then

$$1 + r = \frac{1+i}{1+\pi} \qquad \text{(accurate formula)}$$

Taking log:

$$r = i - \pi \qquad \text{(approximate)}$$

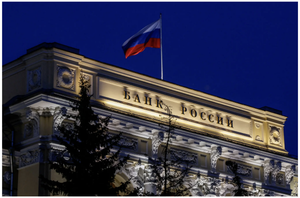
# Factors Influencing Interest Rate

- Demand and supply of loanable funds
  - ex., business investments (demand)
  - ex., household saving (supply via banks)
- Government net supply and demand
  - Amount of gov. spending and new bonds issued
- Monetary policy and expected inflation
- Borrower default risk

**Russia Misses Bond Deadline, Signaling Its First Default on Foreign Debt Since 1918**

Economic sanctions that blocked bond payments have strengthened efforts to seal off Moscow from global markets.

Give this article




The Russian Central Bank headquarters in Moscow. The head of the bank, Elvira Nabiullina, has said there won't be any immediate consequences of a default. Maxim Shemetov/Reuters

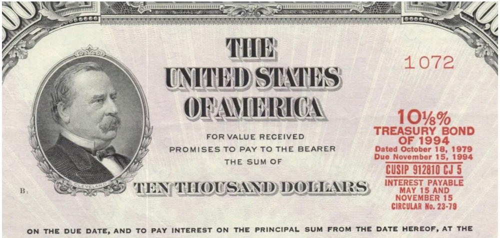
# Risk-Free Interest Rate

- **Risk-free rate**: rate of return of investments which carry no risk
- Approx by the rate of return of bonds issued by government (with no risk of default)
  - T-Bill rate in US dollar
  - SOFR (Secured Overnight Financing Rate)
- Risk-free rate, $r_f$, works as a benchmark when analyzing returns of other (risky) assets

# Computing Interest Payment

## Compounding, Annual Percentage, and Effective Annual Rate

- Interest rate is often specified in annual percentage rate (APR)




- But may be credited (charged) more often than annually
  - Bank accounts: daily
  - Mortgages/leases: monthly
  - Bonds: semiannually in the US (annual in many other countries)
- Effective annual rate (EAR) may differ from APR

# Computing Interest Payment

## Compounding, Annual Percentage, and Effective Annual Rate

**Compounding**: how to compute EAR from APR

- $r$ = APR
- $n$ = periods of compounding (frequency)
- $r/n =$ per-period rate
- EAR is then

$$\text{EAR} = \left(1 + \frac{r}{n}\right)^n - 1$$

# APR vs. EAR: Example 1

## Certificate of Deposit

HSBC's one-year USD term deposit offers annual rate 5% with semi-annual compounding. You've invested \$10,000.
(i) How much money you obtain one year later?
(ii) What is the EAR? Is it different from APR?

- Semi-annual (6-month) rate is $r_6 = 0.05/2 = 0.025$
- After 6 month: Balance = $\$10,000 \times 1.025 = \$10,250$
- 1 year: Balance = $\$10,250 \times 1.025 = \$10,506.25$
- EAR = 5.0625% > 5% = APR
  - $\$10,000 \times (1 + \frac{r}{n})^n$ with $n = 2$

# APR vs. EAR: Example 2

**Car Loan**

Car loan: "Financial charge on the unpaid balance, compounded daily, at the rate of 6.75% per year." You've borrowed \$10,000.
(i) How much money you owe in one year?
(ii) What is the EAR? Different from APR?

- Daily rate is $r_{365} = 6.75/365 = 0.0185\%$
  - Day1: Balance = $\$10,000 \times 1.000185 = 10,001.85$
  - Day2: Balance = $\$10,001.85 \times 1.000185 = 10,003.70$

$$\vdots$$

  - Day 365: Balance = $\$10,696.26 \times 1.000185 = 10,698.24$
- Effective Annual Rate = 6.9824% > 6.750% = APR
- $\$10,000 \times (1 + \frac{r}{n})^n$ with $n = 365$

# Continuous Compounding

## What if interest is paid at ALL points in time?

- Continuous compounding means very large $n$
  - Any contracts compounded continuously?
- Computed by setting $n \to \infty$:

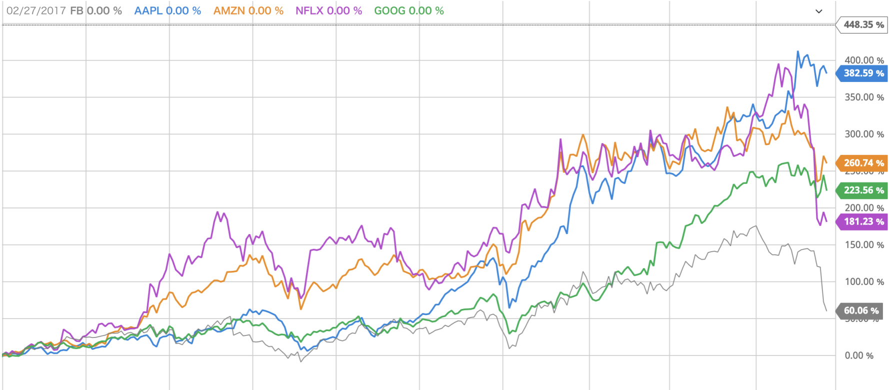
$$\text{EAR} = \lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n - 1 = \exp(r) - 1$$

- Makes equations much simpler
- EAR is increasing or decreasing in $n$? Why?
  - Power of compounding

# Rate of Return of Asset Holdings:
# How They Vary?

# Holding Return and Stock Price

- Facebook, Apple, Amazon, Netflix, Google (5y)
- How would you compare?



# Holding Period Return

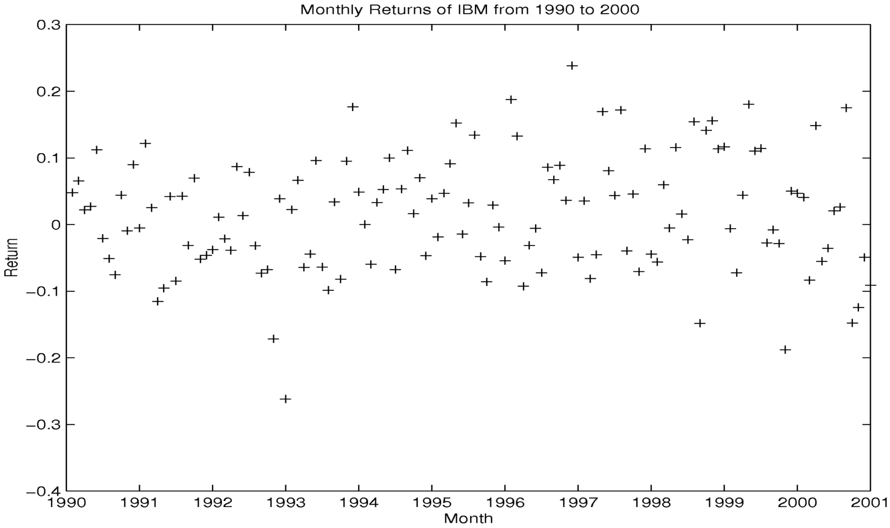
If you buy a stock now and sell it tomorrow, Holding Period Return is

$$\text{HPR} = \frac{P_1 + D_1 - P_0}{P_0}$$

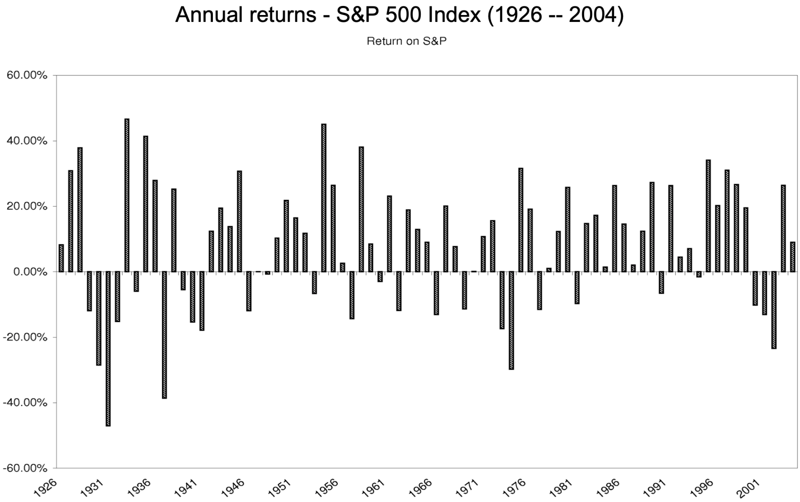
$P_0$: purchasing price, $P_1$: ending price, $D_1$: dividend during this period

- Uncertainty?

# Monthly Return - IBM (1990-2000)



# Annual Return - S&P 500 (1926-2004)



# Expected Return: Formal Definition

- $s$: future state of the economy
  - Assume $s \in \{s_1, s_2, \cdots, s_N\}$
  - e.g., $s_1$ = upturn, $s_2$= downturn, $s_3$ = crisis, etc.
- $p(s)$: probability that state $s$ happens
- $r(s)$: return of the asset when state $s$ happens. E.g.,
  - Johnson & Johnson in pandemic, $r(\text{pandemic})$ = very high
  - Cathay in pandemic, $r(\text{pandemic})$ = very low

### Expected returns

The **expected return** is defined as the weighted average:

$$\mu \equiv \mathbb{E}[r] = \sum_{s=s_1,\cdots,s_N} p(s) r(s)$$

- "First-order moment" of random variable

# Expected Return: Example

| State | very good | good | bad | very bad |
|---|---|---|---|---|
| **Probability** | 0.05 | 0.4 | 0.4 | 0.15 |
| **Return** | 20% | 5% | 1% | -10% |

- The expected return is 1.9%

$$\begin{aligned}\mu &= 0.05 \times 0.2 + 0.4 \times 0.05 + 0.4 \times 0.01 + 0.15 \times (-0.1) \\ &= 0.019\end{aligned}$$

# Variance and Standard Deviation of Return

## Variance

**Variance** of $r(s)$ is defined as

$$\begin{aligned}\sigma^2 &\equiv \mathbb{E}[(r(s) - \mu)^2] \\ &= \sum_s p(s)(r(s) - \mu)^2.\end{aligned}$$

- Expectation of *squired deviation* of $r(s)$ from its mean
  - "Second-order moment" of r.v.
- **Standard Deviation (SD)** is $\sigma$ (= $\sqrt{\text{Variance}}$)
  - Also called volatility

# Variance and Standard Deviation: Example

| State | very good | good | bad | very bad |
|---|---|---|---|---|
| **Probability** | 0.05 | 0.4 | 0.4 | 0.15 |
| **Return** | 20% | 5% | 1% | -10% |

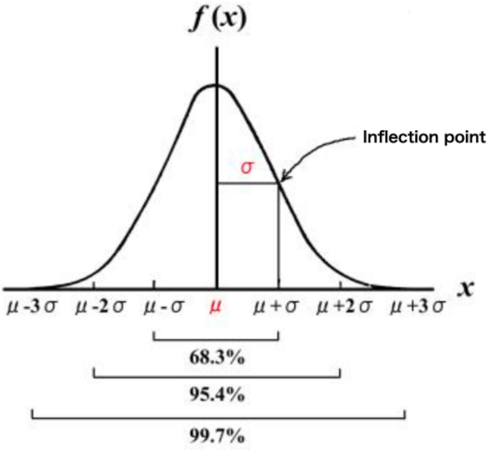
- The expected return is $\mu = 1.9\%$
- Then

$$\begin{aligned}\sigma^2 =& 0.05 \times (0.2 - 0.019)^2 + 0.4 \times (0.05 - 0.019)^2 \\ &+ 0.4 \times (0.01 - 0.019)^2 + 0.15 \times (-0.1 - 0.019)^2 \\ =& 0.00418\end{aligned}$$

- Variance is $\sigma^2 = 0.42\%$ and SD is $\sigma = 6.5\%$

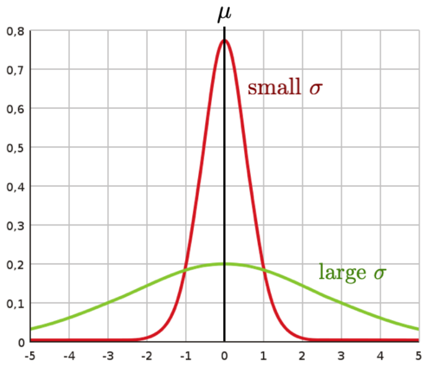
# Mean and Variance

- With normal distribution:



# Mean and Variance

- Large $\sigma \Rightarrow$ realized values are more dispersed around mean
- $\sigma = 0 \Rightarrow \mu$ happens for sure.
- With large $\sigma$, it is hard to predict



# Expectation vs. Realized Returns

From the current perspective:

- Expected Return ($\mu$): forward-looking, what you expect to happen on average in the future
- Risk ($\sigma^2, \sigma$): how volatile you expect the asset return will be
- Assuming that we know distribution of return and state (do we?)

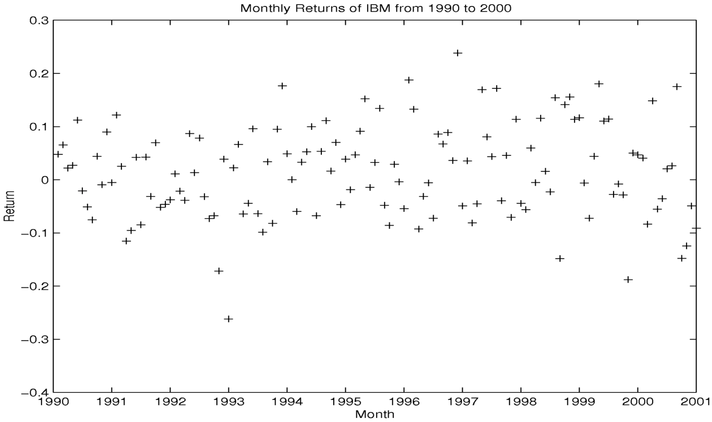
From the ex-post perspective:

- Realized return and risk: backward looking, what did happen
- By observing a large sample size, learn unknown distribution ($p(s)$ and $r(s)$), then use it to extrapolate the future.

# Bringing Data

Suppose that you observe time-series of stock return for $n$ periods

$$\boldsymbol{r}_{\text{data}} = (r_1, r_2, \cdots, r_n)$$



# Bringing Data

Suppose that you observed time-series of stock return for $n$ periods

$$\boldsymbol{r}_{\text{data}} = (r_1, r_2, \cdots, r_n)$$

- The expected return is approximated by the time-series average return
  $\Rightarrow$ **Sample mean** is given by

$$\bar{r} \equiv \frac{1}{n} \sum_{t=1}^{n} r_t$$

- Variance is approximated by expected value of squared deviations
  $\Rightarrow$ **Sample variance** is

$$\hat{\sigma}^2 = \frac{1}{n-1} \sum_{t=1}^{n} (r_t - \bar{r})^2.$$

**Sample standard dev**. is

$$\hat{\sigma} = \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (r_t - \bar{r})^2}$$

# Beyond Mean and Variance

## Third- and fourth-moments

- **Skewness**: how "skewed" the distribution is

$$\text{skewness} \equiv \frac{\mathbb{E}[(r(s)-\mu)^3]}{\sigma^3}$$

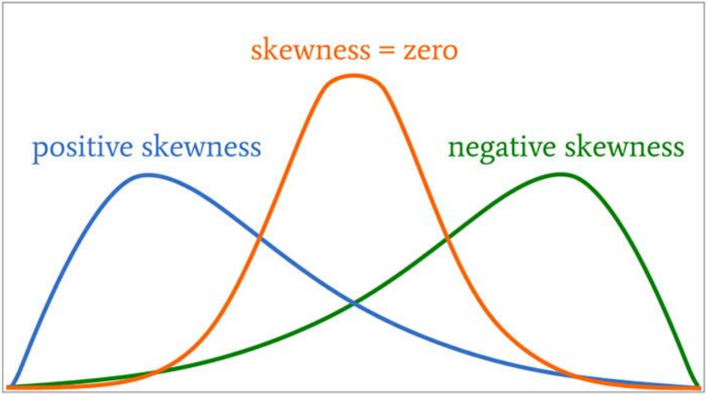
  - Positive skw: rare big gains and frequent small losses
  - Negative skw: rare big losses and frequent small gains

- **Kurtosis**: how much weight is in the tails of a distribution relative to the normal distribution:

$$\text{kurtosis} = \frac{\mathbb{E}[(r(s)-\mu)^4]}{\sigma^4} - 3$$

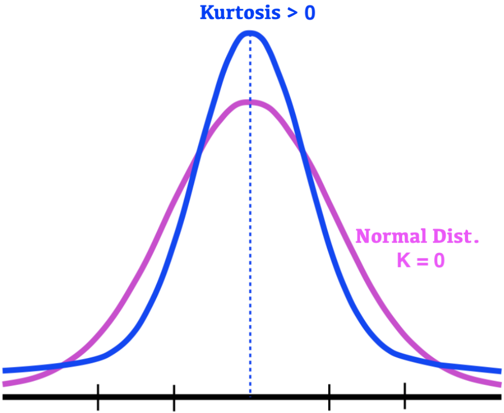
  - High kurtosis: SD underestimates extreme events.

# Skewness




- Normal distribution has zero skewness

# Kurtosis




- High Kurtosis: distribution has fat tail, i.e., rare disaster is likely to happen
- Normal distribution has kurtosis = 0

# Mean-Variance Investors

## Assumptions on investor preference

Investors care about return and risk, called mean-variance preference

- Higher mean return (or expected return) is preferred: $\mu$
- Higher variance (and SD) of return is disliked: $\sigma^2$ and $\sigma$
- Investors care only about risk and return (no higher-order moments)

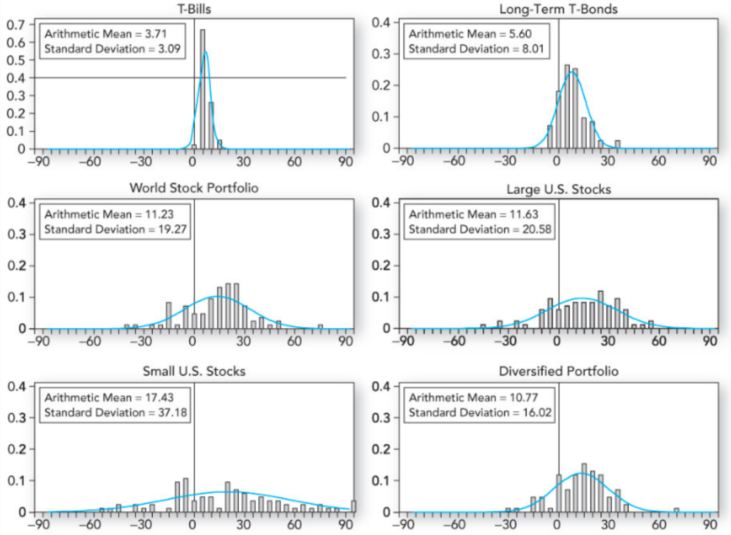
What do risk and return look like? What measures should we use?

# Mean-Variance Relationship

## Higher return comes with higher risk

# Mean-Variance Relationship

## Higher return comes with higher risk



# Measure 1: Risk Premium

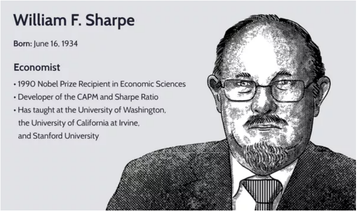
- **Risk premium:** required return (reward) for taking risk:

$$\text{RP} = \mu - r_f = \mathbb{E}[r] - r_f$$

- Forward looking measure
  - A theoretical measure and not observable
- Depends on how investors evaluate risk in conjunction with returns

# Measure 2: Sharpe Ratio

## Measure of risk vs. return




- A measure of risk-return tradeoff
  - Willian Sharpe 1990 Nobel Laureate

### Sharpe Ratio

Defined by

$$\text{SR} = \frac{\mu - r_f}{\sigma} = \frac{\text{Expected return} - \text{risk-free rate}}{\text{Standard Dev.}}$$

- $\mu - r_f = \mathbb{E}[r] - r_f$: risk premium

- Higher Sharpe ratio is better than lower ones
- Important tool to determine optimal portfolio (class 5 $\sim$)

# Sharpe Ratio: Example

## Measure of risk vs. return

### Sharpe Ratio

Suppose that two stocks are available: AAPL & AMZN. AAPL has historical mean return of 6.5% and SD is 8%. AMZN has mean return of 7.8% with SD 20%. Risk-free rate is at 4.3%. If you are mean-variance investor, which stock you should hold?

$$\mathrm{SR}_{AAPL} = ??$$
$$\mathrm{SR}_{AMZN} = ??$$

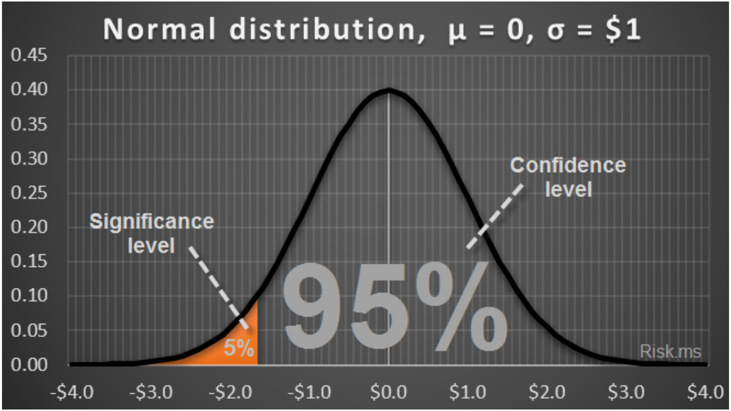
- If you invest using SR, which one is better?

# Measure 3: Value at Risk (VaR)

- VaR: measures the risk of losing money in investments
- VaR is the $p^{\text{th}}$ percentile of the distribution
  - typically $p = 5$ or $p = 10$ (let's use $p = 5$)
- There is a 5% prob. that the return/payoff dips below VaR
- Ex: "VaR = \$1,000" $\Rightarrow$ "prob. that the payoff is below \$1,000 is 5%"

# Measure 3: Value at Risk (VaR)

## With Normal distribution



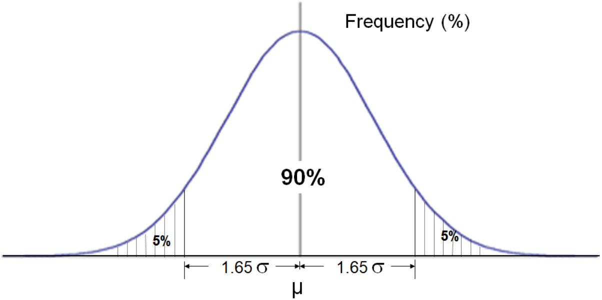

- 95% of time, you obtain return/payoff higher than VaR
- Equivalently, returns lower than VaR happens with 5%

# VaR with Normal Return

- With time-series data, you can compute VaR
  - Expected return $\mu$ is approx. by $\bar{r} = \sum r_t / n$
  - SD $\sigma$ is approx. by $\hat{\sigma} = \sqrt{\sum (r_t - \bar{r})^2 / (n-1)}$
- If $r_t$ follows Normal dist.

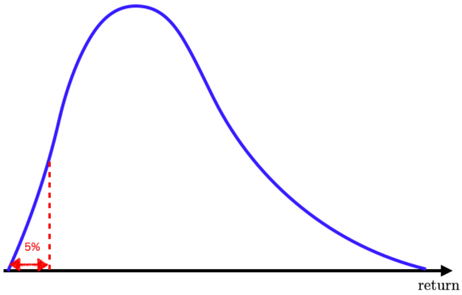
$$\text{VaR} = \bar{r} - 1.65\hat{\sigma} \tag{1}$$

  - (-1.65 = 5% of standard normal distribution; use -2.33 for 1%)



# VaR with Non-Normal Return

- Realized returns often exhibits “fat tails”
  - Normality assumption is violated
  - Formula (1) no longer holds
- Mean and variance no longer fully capture risk-return relationship
- But, VaR is still informative without knowing true distribution
- Can be numerically computed with data
  - e.g., Excel function `PERCENTILE(A1:An,p)`



# Four Facts about Risk and Return

- Real risk-free interest rate has been slightly positive on average (why?)
- Return on more risky assets has been higher (on average) than that on less risky assets (why?)
- Returns on risky assets can be highly correlated to each other (why?)
- Returns on risky assets are usually serially uncorrelated (why?)

# Next Class

- How investors evaluate risk and return?